Rolling D and D Dice

This week's [Fiddler on the Proof](#) (12 January 2024) asks:

> Two people are sitting at a table together, each with their own bag of six "DnD dice": a **d4**, a **d6**, a **d8**, a **d10**, a **d12**, and a **d20**. Here, "**d*x***" refers to a die with *x* faces, numbered from 1 to *x*, each with an equally likely probability of being rolled.
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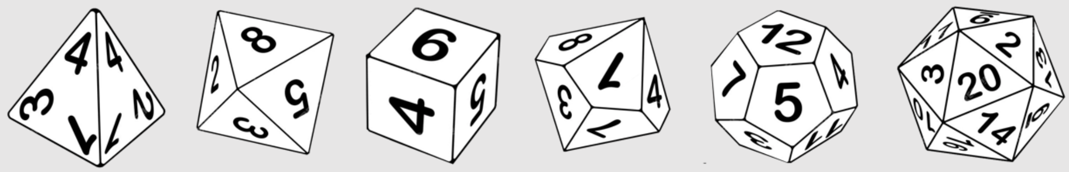
> 
>
> Both people randomly pick one die from their respective bags and then roll them at the same time. For example, suppose the two dice selected are a **d4** and a **d12**. The players roll them, and let's further suppose that both rolls come up as 3. What luck!
>
> What's the probability of something like this happening? That is, what is the probability that both players roll the same number, whether or not they happened to pick the same kind of die?

The easiest way to approach this is by looking at each number in turn. Let's start with 1.

Chance of Rolling a 1

What's the probability that a player rolls a 1? There are six ways it could happen:

Step 1	Prob-ability	Step 2	Prob-ability	Combined probability	
Pick die **d4**	1/6	then roll 1	1/4	1/24	= .041667
Pick die **d6**	1/6	then roll 1	1/6	1/36	= .027778
Pick die **d8**	1/6	then roll 1	1/8	1/48	= .020833
Pick die **d10**	1/6	then roll 1	1/10	1/60	= .016667
Pick die **d12**	1/6	then roll 1	1/12	1/72	= .013889
Pick die **d20**	1/6	then roll 1	1/20	1/120	= .008333
Total					**.129167**

So a player has about a 12.9% chance of rolling a 1.

Likewise with numbers 2, 3, and 4 since they appear on exactly the same dice as number 1.

Chance of Rolling a 5

What about rolling a 5? Things are different because **d4** does not have a 5 on it. The first line of the previous table no longer counts:

Step 1	Prob-ability	Step 2	Prob-ability	Combined probability	
Pick die **d4**	1/6	then roll 5	0	0	= .000000
Pick die **d6**	1/6	then roll 5	1/6	1/36	= .027778
Pick die **d8**	1/6	then roll 5	1/8	1/48	= .020833
Pick die **d10**	1/6	then roll 5	1/10	1/60	= .016667
Pick die **d12**	1/6	then roll 5	1/12	1/72	= .013889
Pick die **d20**	1/6	then roll 5	1/20	1/120	= .008333
Total					**.087500**

A player has 8.75% chance of rolling 5. (Likewise with 6.)

Chance of Rolling a 7

The number 7 appears only on **d8**, **d10**, **d12**, and **d20**.

Step 1	Prob-ability	Step 2	Prob-ability	Combined probability	
Pick die **d4**	1/6	then roll 7	0	0	= .000000
Pick die **d6**	1/6	then roll 7	0	0	= .000000
Pick die **d8**	1/6	then roll 7	1/8	1/48	= .020833
Pick die **d10**	1/6	then roll 7	1/10	1/60	= .016667
Pick die **d12**	1/6	then roll 7	1/12	1/72	= .013889
Pick die **d20**	1/6	then roll 7	1/20	1/120	= .008333
Total					**.059722**

A player has 5.97% chance of rolling 7. (Likewise with 8.)

All Numbers

Continuing this way, we can find the probability of rolling every number from 1 to 20. [Table A.]

Number	Chance of rolling it
1	.129167
2	.129167
3	.129167
4	.129167
5	.087500
6	.087500
7	.059722
8	.059722
9	.038888
10	.038888

Number	Chance of rolling it
11	.022222
12	.022222
13	.008333
14	.008333
15	.008333
16	.008333
17	.008333
18	.008333
19	.008333
20	.008333
Total	**1.000000**

Both Players Rolling the Same Number

What are the chances that both players roll a 1? For this to happen, Player A must roll a 1 (0.129167 probability), then Player B must roll a 1 (0.129167 probability). The combined probability is

$$0.129167 \times 0.129167 = 0.016684$$

We can similarly calculate the probability of both players rolling any of the other numbers. The following table shows all these probabilities. Note, it is the same as the previous table except that all the entries in it are squared. [Table B.]

Number	Chance that both players roll it	Number	Chance that both players roll it
1	.016684	11	.000494
2	.016684	12	.000494
3	.016684	13	.000069
4	.016684	14	.000069
5	.007656	15	.000069
6	.007656	16	.000069
7	.003567	17	.000069
8	.003567	18	.000069
9	.001512	19	.000069
10	.001512	20	.000069
		Total	**.093750**

Add up all the ways of rolling equal numbers and you get **9.375 percent**. And that's the answer to the puzzle.

Extra Credit

Instead of two people sitting at the table, now suppose there are *three*.

Again, all three randomly pick one die from their respective bags and roll them at the same time. For example, suppose the three dice selected are a d4, a d20, and a d12. The players roll them, and let's further suppose that the d4 comes out as 4, the d20 comes out as 13, and the d12 comes out as 4. In this case, there are *two* distinct numbers (4 and 13) among the three rolls.

On average, how many *distinct* numbers would you expect to see among the three rolls?

We have already computed Table A above which contains the probability of rolling each specific number from 1 to 20. In the code below, we'll access the entries of this table as

```
table[number]
```

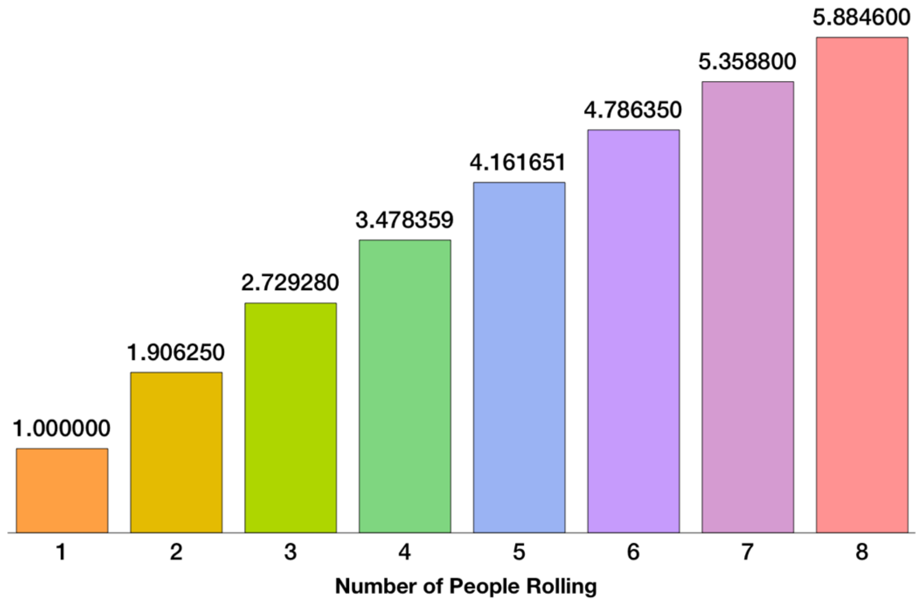
The following code goes through every possible combination of the three numbers rolled, makes a note of how many distinct numbers there are in each combination, calculates the probability of that combination happening, and adds up each contribution to the overall average.

```
var distinctAverage = 0.0
for numberA in 1...20
    {
    for numberB in 1...20
        {
        for numberC in 1...20
            {
            let distinctCount = Set( [ numberA, numberB, numberC ] ).count
            let prob = table[numberA] * table[numberB] * table[numberC]
            distinctAverage += prob * distinctCount
            }
        }
    }
print( "Distinct average =", distinctAverage )
```

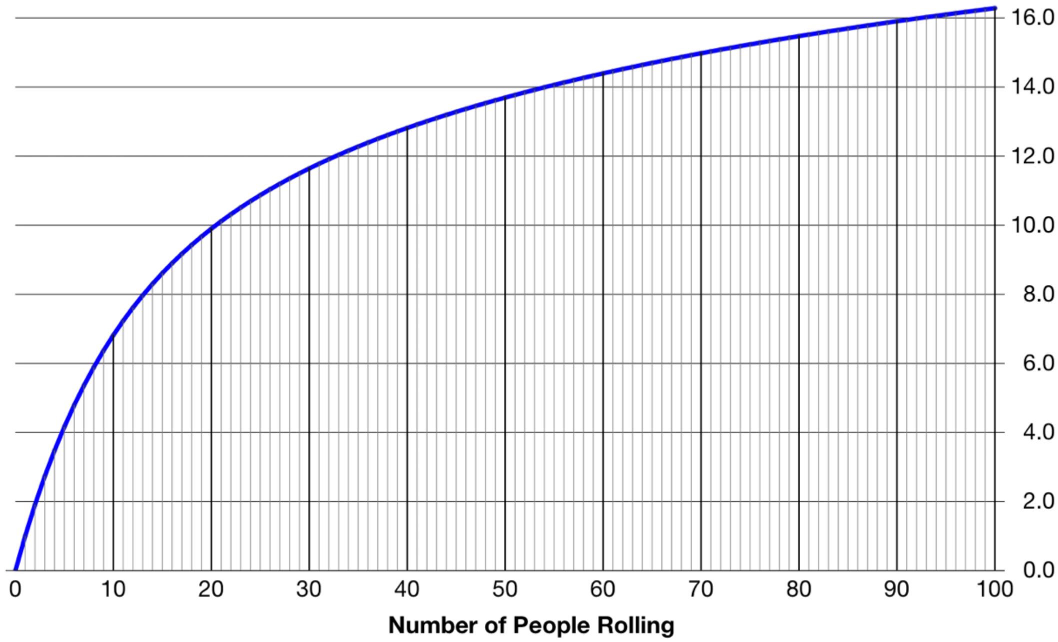
Running this code produces an answer of **2.729280**.

More People

Continuing in the same vein, here is the number of distinct rolled numbers you can expect to see when *n* people are rolling:



And here's a plot with more people:



Even with 100 different players, you will likely see only about 16 or 17 different numbers rolled among them. It takes 250 players to have an expected number of different rolls of more than 19.

With 1000 players, the expected number of different rolls is 19.998.