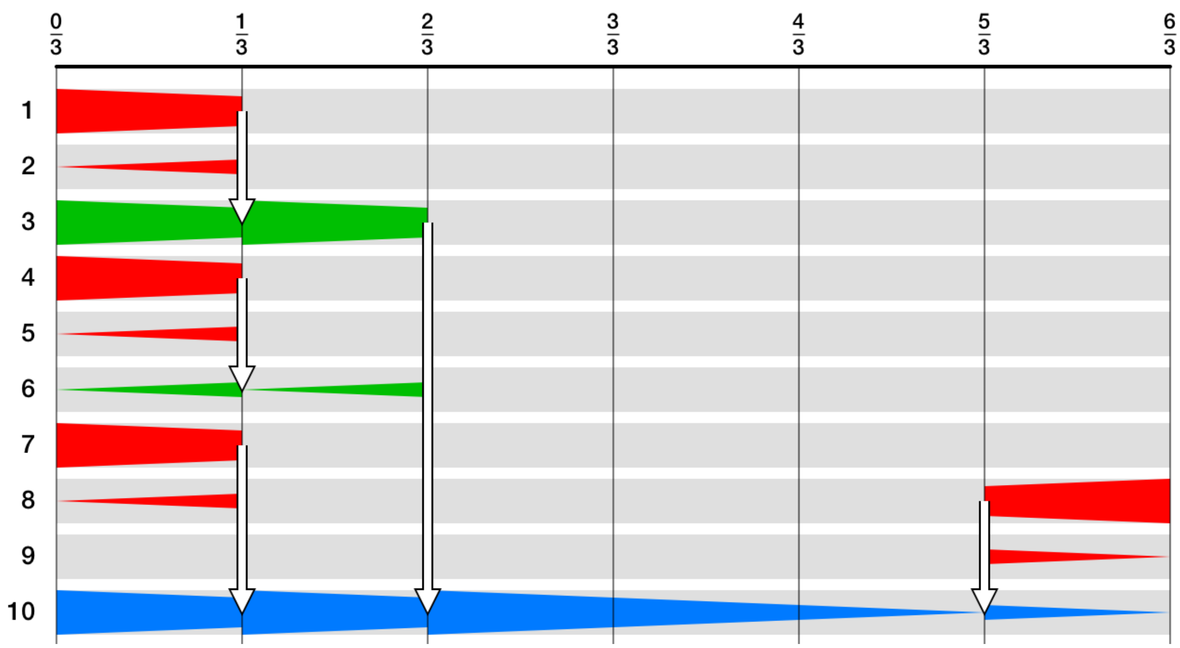
Scenic Route

This week's Fiddler on the Proof (8 March 2024) asks:

> There is a circular road, along which travelers can drive in either direction. However, there is only one gas station on the loop. Driving the full loop in your car requires 40 gallons of gas, but your car's fuel tank has a maximum capacity of 20 gallons. That said, you'd love to see every last spot along the route.
>
> Of course, you can't achieve this with just your own car. Fortunately, you can call on any number of your Fiddler Nation friends, all of whom happen to have the same make and model car as you, each with a 20-gallon fuel tank and identical fuel efficiency.
>
> Now, all the cars, including yours, *must* start and end at the gas station. However, only *your* car must cover the entire route. The gas station can be visited (and refueled at) by any car, any number of times. Cars may also transfer fuel from one to another, provided they meet up together at a spot along the route.
>
> What is the smallest number of cars (including yours) needed for you to see every spot on the circular route?
>
> **Extra Credit**
>
> What is the minimum amount of *gas* collectively needed by all cars for this journey? (Remember, all the cars must begin *and* end at the gas station.)

Here is the first idea I came up with. I don't know if there is a better way.



There are three cars: a red car, a green car, and a blue car. The numbers across the top of the chart show how far along the course the cars have proceeded, measured in tanks of fuel needed to get to that point. It takes 6/3 (two) tanks of fuel for one car to travel the whole course.

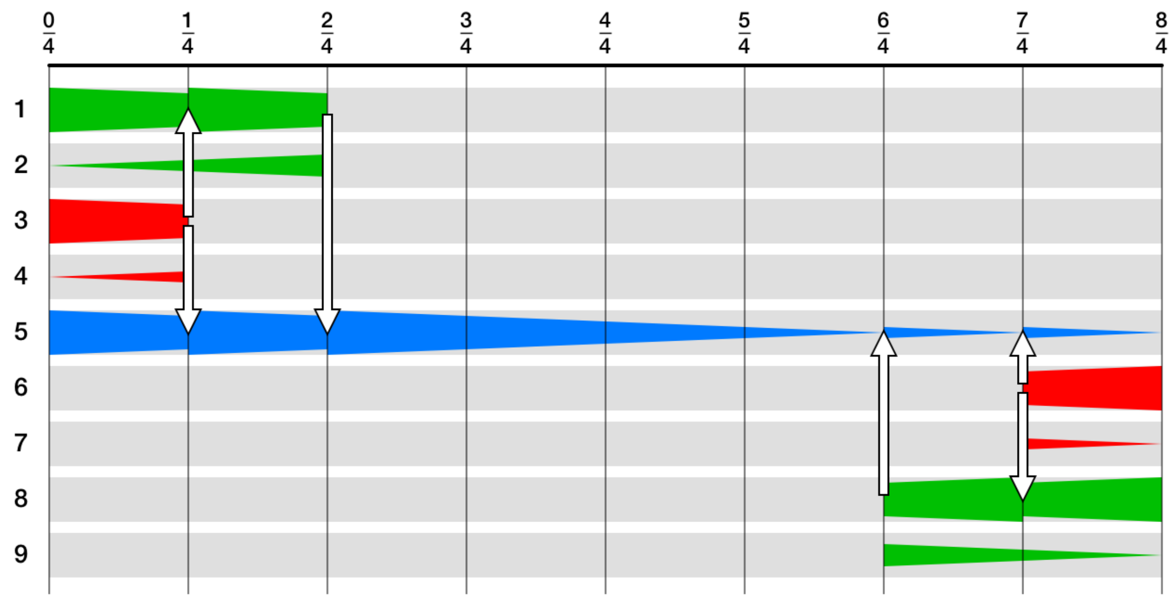
In row 1 of the chart, the red car uses 1/3 of a tank to drive one leg of the course. In row 3, the green car does the same thing. At that point the red car transfers 1/3 of a tank to the green car (a white arrow shows the transfer). The red car still has 1/3 of a tank left, which is enough to get him back home, in row 2.

The blue car is the one that travels the entire route, shown in row 10. After the first leg, the blue car picks up 1/3 of a tank from the red car. After the second leg, he picks up another 1/3 of a tank from the green car. That gives him enough fuel to get to the 5/3 point on the course where he runs out of gas. But the red car goes backwards from the gas station to meet the blue car and transfers 1/3 of a tank of fuel to him which is enough to get him home.

How much fuel was consumed in all? Count up the legs traveled: the red car goes 8 legs, the green car goes 4 legs, and the blue car goes 6 legs. That's a total of 18 legs. Each leg represents 1/3 of a tank, so together the cars use 18/3 = 6 tanks of fuel.

Dave Moran's Solution

Dave Moran posted a more fuel efficient solution on Twitter:



All three cars start off together. At the 1/4 mark, the red car transfers 1/4 tank to each of the green and blue cars, leaving red with 1/4 of a tank to get back home.

The green car, with a full tank again, now has enough capacity to get to the 2/4 mark, transfer 1/4 tank to the blue car, and get back home.

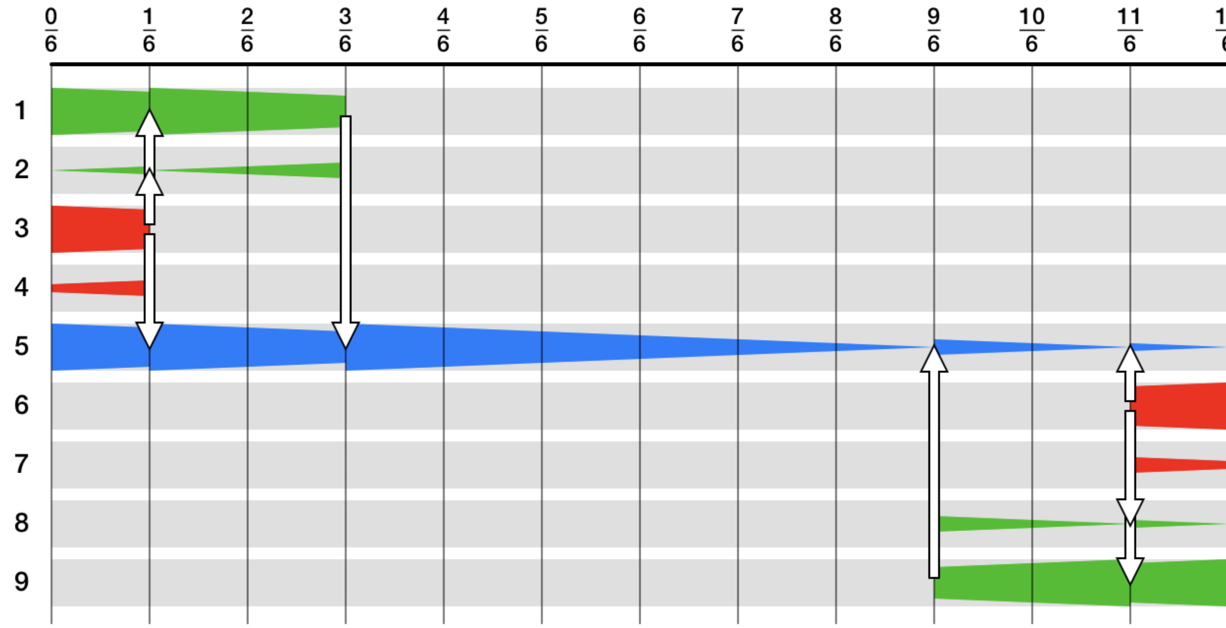
The blue car, having been topped up twice, can now make it to the 6/4 mark before he runs out of gas.

The green car, with some help from the red car, drives backwards from the gas station to give blue enough fuel to reach the 7/4 mark. Then the red car supplies blue with enough gas to reach home.

In all, the red car travels 4 legs, the green car travels 8 legs, and the blue car travels 8 legs, for a total distance of 20 legs. Each leg uses 1/4 of a tank, so the cars use a total of 5 tanks of gas.

Sanandan Swaminathan's Solution

Sanandan Swaminathan, the submitter of the puzzle, has an even *more efficient* solution! Here is a slightly modified description of Sanandan's method:



All three cars start off together. At the 1/6 mark, the red car transfers 1/6 tank to each of the green and blue cars. Then red waits where he is until the green car comes by on his way home and red transfers another 1/6 to green. Even after giving away half a tank of fuel, red has more than enough gas to make it back home.

The green car, with a full tank at the 1/6 mark, advances to the 3/6 mark and transfers 1/3 of a tank to blue. Green now has 1/3 of a tank left, which is enough to get him back to the 1/6 mark where red is waiting. Green gets 1/6 tank of fuel from red and now has enough to make it back home.

Meanwhile, the blue car, having been topped up twice, is sitting on the 3/6 mark with a full tank of gas. That means blue has enough fuel to drive from 3/6 to 9/6, where he runs out of gas

The green car, with some help from the red car, drives backwards from the gas station to meet blue at the 9/6 mark and transfers 1/3 of a tank of fuel to him. Red then meets blue at the 11/6 mark and transfers another 1/6 of a tank of fuel, which is enough for blue to get home.

(The coordination between red and green on the back end is the mirror image of their coordination on the front end.)

You can see visually on the chart how much fuel this method uses. Red uses a total 4/6 tanks of fuel, green uses 12/6 tanks, and blue uses 12/6 tanks. In total, the cars use 28/6 tanks of fuel, or 4⅔ tanks. That's one third of a tank less than Dave Moran's method.