

**Money Cube Action
in Low-Wastage Positions**

By: Michelin Chabot

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PREFACE

To obtain a proper "Money Cube Action in Low-Wastage Positions", so far there were several approaches that were proposed. An approach is defined as an ensemble of criteria that allows if a player should double or not, redouble or not, take the cube or pass it.

The oldest approach, the "8%, 9%, 12%", was proposed in the 70s.

The second oldest approach was proposed by Thorp. This approach was also proposed in the 70s. This approach was more precise than the "8%, 9%, 12%" approach.

Afterwards, there were a few other "improved approaches" proposed. Some of those new improved approaches were developed to analyze positions with wastage. The better known are: Ward, Keeler/Gillogly, Lamford/Gasquoine, and Keith. The previous list isn't necessarily exhaustive.

Recently in 2004, in the book "Backgammon BOOT CAMP", Mr Walter Trice proposed another approach. His approach seems to be more precise than all the previous ones.

Even if the authors of certain approaches seem to pretend, directly or indirectly, that their approach is the best one, I was always sceptical and I always asked myself the two following questions:

1) Is it possible to evaluate the precision of an approach the objective of which is to obtain a proper "Money Cube Action in Low-Wastage Positions"?

2) If yes, what is the best approach?

To try to answer the above questions, I began to do some research to verify if the existing theories had already answered these questions, but I found nothing. So, I tried to find answers to my own questions. I tried several analysis techniques. Having tried some, I finally managed to identify the one which is probably the right one. This technique consists of only 2 steps. The first step consists of developing what I call the optimal approach. The second step consists of taking the obtained results for each proposed approach and comparing them to the results obtained by using the optimal approach. To obtain the desired results, I had to develop some previously unpublished analysis techniques. I also developed a new approach. So, the main purposes of this article are to explain the technique that I used and to propose a new approach.

If you have any comments to formulate to improve this article, contact me: michelinchabot@gmail.com.

Michelin Chabot

INTRODUCTION

The general purpose of this article is to elaborate the doubling cube theory in money games, for running positions, in which there is little or no wastage.

The specific purposes of this article are:

- To present the optimal approach.
- To analyze three known approaches proposed so far.
- To propose a new approach.

This article includes 3 parts:

- Part 1 entitled: *"The optimal approach"*, begins by giving the definitions and explanations of all the concepts that will be used in this article. This first part also presents in great detail, the technique used to develop the optimal approach. This part was written to allow a skeptical reader to be able to verify this technique, and to be able to confirm that the obtained approach is really the optimal one.
- Part 2 entitled: *"Analysis of some approaches"*, analyzes three known approaches proposed so far, namely: the 8%, 9%, 12% approach, Thorp's approach and Trice's approach. This part also proposes a new approach, namely: the Chabot one.
- Part 3 entitled: *"From theory to practice"*, mainly explains, with the help of a few practical examples, how to use the recommendable approaches.

Part 1: The optimal approach

1.1 Generalities

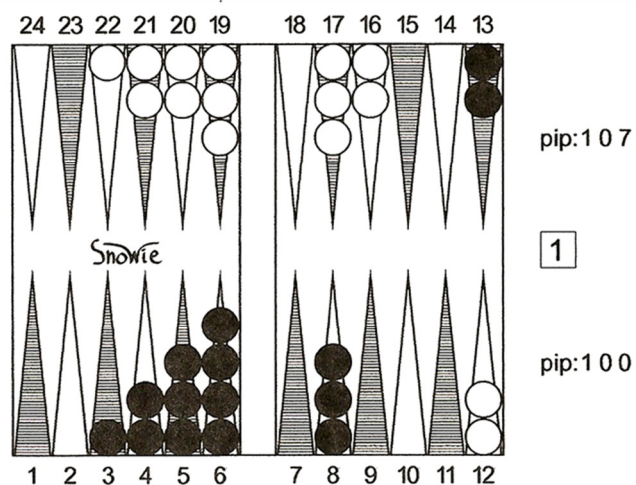
When a player, whose turn it is to play, has the advantage in a running position (also called a race), this means that he is the favorite player to win the game, but this does not necessarily mean that he should double or redouble. When your opponent has the advantage in a running position and he doubles you or he redoubles you, then you have to choose between take or pass. To help you to decide when you should double, redouble, take or pass, it is obviously preferable to use mathematical criteria, which is equivalent to saying that you should use an approach.

In part 1 of this article, we will begin by defining what a running position is, and what a *"Low-wastage position"* is. Afterwards, we will give the definitions of the terms and the concepts used in this article. And finally, we will develop in great detail, how *"the optimal approach"* was obtained.

1.1.1 Pipcount

The *"pipcount"* is the means use to calculate, and to evaluate a running position. A checker on the 1-point represents 1 pip. A checker on the 2-point represents 2 pips, etc... a checker on the 24-point, represents 24 pips and a checker on the bar represents 25 pips. The pipcount is obtained by adding the number of pips associated to every checker. Here is an example:

Position No. 1



For Black, the checker on the 3-point represents 3 pips. The 2 checkers on the 4-point represent 4 pips each. The 3 checkers of the 5-point represent 5 pips each, etc. Therefore, Black's pipcount is obtained as follows:

1 x 3 pips	=	3 pips
2 x 4 pips	=	8 pips
3 x 5 pips	=	15 pips
4 x 6 pips	=	24 pips
3 x 8 pips	=	24 pips
2 x 13 pips	=	<u>26 pips</u>
Total	=	100 pips

The same technique must be applied for White, but taking into account that the Black's 24-point corresponds to White's 1-point, etc. Therefore, White's pipcount is obtained as follows:

1 x 3 pips	=	3 pips
2 x 4 pips	=	8 pips
2 x 5 pips	=	10 pips
3 x 6 pips	=	18 pips
3 x 8 pips	=	24 pips
2 x 9 pips	=	18 pips
2 x 13 pips	=	<u>26 pips</u>
Total	=	107 pips

Thus, for position no.1, the obtained pipcount is: Black = 100 pips, White = 107 pips.

1.1.2 Running positions

The running positions can be separated in two groups, namely:

- The positions in which there is contact.
- The positions in which there is very little or no contact.

Generally speaking, a running position (or a race) is a position in which there is no contact.

When there is contact in a running position, the strategy the favorite player can use consists of breaking contact to obtain a running position without contact. The underdog should use the reverse strategy. Indeed, the underdog should not try to break contact. He should rather try to play to maximize the contact. Indeed by maximizing the contact, he maximizes his chances to hit a blot. Sooner or later, all running positions in which there are contacts will inevitably be transformed into a running position without contact.

This article analyzes running positions in which there is very little or no contact. Please note that when there is still some contact and when the probability of hitting a blot is very weak (for example, around 1%), then for all intents and purposes, these kind of positions could also be considered as being a running position without contact.

1.1.3 Low-wastage positions

Running positions, in which there is very little or no contact, can in turn be separated in two groups, namely:

- *Low-wastage positions*
- *Positions with wastage*

"Low-wastage positions" are positions where the checkers are well distributed and that have very few or no weak points. To handle the cube for this type of position, you can use an approach based on a ***"raw pipcount"***, meaning a pipcount obtained without any adjustment. Once you get the raw pipcount, it is necessary to use an approach that was developed for such a position.

To handle the cube for this type of position, there are several approaches which were already proposed. Among them are the 8%, 9%, 12% approach, Thorp's approach and Trice's approach. These three (3) approaches will be analyzed in part 2 of this article.

"Positions with wastage" are positions in which certain elements of the checker distribution contain one or several weak points. To handle the cube for this kind of position, it is necessary to proceed in 3 steps:

- 1) The first step consists of calculating the "raw pipcount".
- 2) The second step consists of calculating the "adjusted pipcount". For this, it is necessary to make some adjustments according to certain criteria which are based on certain weak points.
- 3) The third step consists of using the obtained "adjusted pipcount" and using an approach which was developed to handle the cube for a low-wastage position.

To make pipcount adjustments, several approaches were proposed. Among others, they are the four (4) following approaches:

- Ward's approach
- Keeler/Gillogly's approach
- Lamford/Gasquoine's approach
- Keith's approach

Tom Keith wrote an article to analyze the four previously mentioned approaches. To consult this article, proceed as follows:

- Go on the site "Backgammon Galore!"
- Choose the following subject: "Articles By Author"
- Choose the author: "Keith, Tom"
- Choose the title: "2004: Cube Handling in Noncontact Positions "

To calculate the adjusted pipcounts for positions with wastage, we will refer to this article and this topic will not be further elaborated since it isn't the goal of this article.

To determine as clearly as possible what a "Low-wastage position" is, you must first determine the nature of the weak points for which adjustments may be necessary. Among all the approaches proposed to calculate the adjusted pipcounts, the most popular, and probably the best one is that of Ward. In the book: "Backgammon BOOT CAMP", and more specifically in the chapter: "Adjusted Pipcount Methods", Mr. Walter Trice explains very clearly how he suggests using this last approach. Here is the appropriate extract:

"Here is how you compute the Ward adjusted pipcounts:

- 1) *Start with the pipcount.*
- 2) *If a player has more checkers on the board than his opponent, add two pips for each extra checker.*
- 3) *If there are more than two men on the ace point, add two pips for the third and each additional checker.*
- 4) *If there are more than two men on the deuce point, add one pip for the third and each additional checker.*
- 5) *If a player has fewer vacant home-board points than his opponent, subtract one pip for each extra-occupied point.*
- 6) *Add ½pip for each extra checker outside the home board.*

The resulting adjusted pipcounts provide a more accurate picture of the race than the raw pipcounts, and fortunately enough, you can derive cube handling from adjusted pipcounts just as easily as you can from the raw."

So, according to Ward's approach, the weak points, which warrants a pipcount adjustment, are the following one:

- A number of checkers on the board, which is different than those of the opponent.
- The presence of 3 checkers or more on the 1-point.
- The presence of 3 checkers or more on the 2-point.
- A number of vacant points (or gaps) on the home board, which is different than those of the opponent.
- The presence of checkers outside the home board, which is different than those of the opponent.

There are other approaches that suggest ways of making pipcount adjustments. As mentioned previously, there is Keeler/Gillogly's approach, Lamford/Gasquoine's approach and Keith's approach. The weak points, which were mentioned in these previous approaches, are the following one:

- The presence of more than 1 checker on the 1-point.
- The presence of more than 1 checker on 2-point.
- The presence of more than 3 checkers on 3-point.
- The presence of more than 5 checkers on some point.
- The presence of a gap on the 2-point.
- The presence of a gap on the 3-point.
- The presence of a gap on the 4-point.
- The presence of a gap on the 5-point.

Based on previous considerations, we can define a "Low-wastage position" as being a position which meets the following 3 conditions:

- 1) There is very little or no contact.
- 2) There is a fairly good checker distribution.
- 3) There are few or no weak points such as:
 - The presence of more than 1 checker on the 1-point.
 - The presence of more than 1 checker on the 2-point.
 - The presence of more than 3 checkers on the 3-point.
 - The presence of more than 5 checkers on some point.
 - The presence of a gap on the 2-point.
 - The presence of a gap on the 3-point.
 - The presence of a gap on the 4-point.
 - The presence of a gap on the 5-point.
 - A number of checkers on the board, higher than your opponent.
 - A number of vacant points (or gaps) on the home board, which is different higher than your opponent.
 - A number of checkers the outside home board, higher than your opponent.

1.1.4 Definitions and explanations of the concepts used

To understand this whole article, it is necessary to define certain terms and to elaborate on certain concepts.

A "*cubeless position*" is a position in which there is no cube. Even if in practice, the cube is always used, please note that to evaluate a position, Snowie used such a concept.

The expression "*to be the favorite*", simply means that in a cubeless position, the probability of winning is higher than 50%. Given that the average number of pips in a throw of the dice is very slightly higher than 8 pips (more exactly 8.1666... pips), this usually means that when there is no contact, if the player who's turn it is to play lags behind by 4 pips, he has an approximate 50% probability of winning. For example, for the following pipcount: Black = 100 pips, White = 96 pips, if Black is on roll, then he has an approximate 50% chance of winning the game. But for the following pipcount: Black = 100 pips, White = 97 pips, Black has around 52% chance of winning the game. Thus for a player whose turn it is to play, to be considered as "being the favorite", he usually must have an advantage in the running position or not be behind by more than 3 pips.

The "*leader*" is the player who meets the following 2 conditions:

- 1) He is the favorite to win a cubeless game.
- 2) It is his turn to play.

The "*trailer*" is the opponent of the leader. So, he is not the favorite player and he has to wait to play.

The abbreviation "*P*" represents the leader's Pipcount. "*P*" is always expressed in pips.

The abbreviation "A" represents the leader's Advantage. The value of "A" is obtained by subtracting the leader's pipcount from the trailer's pipcount. "A" is always expressed in pips. For example, if the leader's pipcount is 100 pips, and if the trailer's pipcount is 107 pips, then "A" = 7 pips (107 pips – 100 pips).

The abbreviation "D" represent the trailer's Disadvantage. The value of "D" is also obtained by subtracting the leader's pipcount from the trailer's pipcount. "D" is always expressed in pips. In fact, the trailer's Disadvantage is equal to the leader's Advantage. So "D" is equal to "A" ($D = A$).

A "*criterion*" is a mathematical formula used for calculating a value. The mathematical formula used to represent a criterion can take on several forms. To obtain a criterion, we need to begin by using a formula, and afterwards, we always have to make an additional mathematical operation such as "rounding up" or "rounding down". For "rounding up" the abbreviation used is "up". For "rounding down" the abbreviation used is "down". Here are some examples of criteria:

- $(P \times 8\%)$, up
- $(P \times 9\%)$, up
- $(P \times 12\%)$, down
- $((P \times 10\%) - 2)$, up
- $((P/10) - 3)$, up
- $(P/8)$, down
- $(P/8)$, up

Note: You must always carry out the mathematical operations according to the order imposed by the brackets. For example, for the criterion " $((P/10) - 3)$, up", the first operation to be made is: " $(P/10)$ ". The second operation to be made is "- 3", and finally the last operation to be made is "up". The obtained result is always expressed in pips without using any decimal.

Here are some examples of values when $P = 100$ pips:

- $((P \times 10\%) - 3)$, up = 7 pips
- $(P \times 8\%)$, up = 8 pips
- $((P \times 10\%) - 2)$, up = 8 pips
- $(P \times 12\%)$, down = 12 pips
- $(P/8)$, down = 12 pips
- $(P/8)$, up = 13 pips

The abbreviation "DP" means: "Doubling Point". DP represents the minimum advantage that the leader must have to double. This means that the cube is in the center. DP is always expressed in pips.

You get a DP value by using a criterion. For example, if the criterion used is: $(P \times 8\%)$, up, then for a P ranging from 98 pips to 102 pips, you get the following DP values:

P	DP
98	8 pips
99	8 pips
100	8 pips
101	9 pips
102	9 pips

If for a specific pipcount, the obtained DP is 8 pips with the criterion you use, then the correct way to handle the cube is as follows:

- If A is 7 pips or less, then the leader must not double.
- If A is 8 pips or more, then the leader must double.

Note: In the previous paragraph, the expression used was "the leader must double" and not "the leader should double" or "the leader could double". This relates to the fact that the goal of this article is to take on a more theoretical nature than a practical one. Obviously, in practice there are always practical considerations the consequence of which would be that the theoretical results could not necessarily be "respected to the letter". The goal of this article isn't to analyze those kinds of considerations, therefore, the word "must" is preferred to the word "should" or "could".

To be as specific as possible, let's suppose that the leader's pipcount is 100 pips and that the obtained DP is 8 pips, then this means that:

- If the trailer's pipcount is 107 pips or less, then the leader must not double.
- If the trailer's pipcount is 108 pips or more, then the leader must double.

The *abbreviation* "**RP**" means: "Redoubling Point". RP represents the minimum advantage that the leader must have to redouble. This means that you are the leader and that you have the possession of the cube. RP is always expressed in pips.

You get a RP value by using a criterion. For example, if the criterion used is: $(P \times 9\%)$, up, then for a P ranging from 98 pips to 102 pips, you get the following RP values:

P	RP
98	9 pips
99	9 pips
100	9 pips
101	10 pips
102	10 pips

If for a specific pipcount, the obtained RP is 9 pips with the criterion you use, then the correct way to handle the cube is as follows:

- If A is 8 pips or less, then the leader must not redouble.
- If A is 9 pips or more, then the leader must redouble.

To be as specific as possible, let's suppose that the leader's pipcount is 100 pips, the leader has the possession of the cube and that the obtained RP is 9 pips, then this means that:

- If the trailer's pipcount is 108 pips or less, then the leader must not redouble.
- If the trailer's pipcount is 109 pips or more, then the leader must redouble.

The *abbreviation "LTP"* means "Last Take Point". LTP represents the maximum disadvantage that the trailer must have to take the cube. When the leader offers you the cube, you must then decide to take or pass. The fact that the cube is in the center, or belongs to the leader, changes absolutely nothing. LTP is always expressed in pips.

You get a LTP value by using a criterion. For example, if the criterion used is: $(P \times 12\%)$, down, then for a P ranging from 98 pips to 102 pips, you get the following LTP values:

P	LTP
98	11 pips
99	11 pips
100	12 pips
101	12 pips
102	12 pips

If for a specific pipcount, the obtained LTP is 12 pips with the criterion you use, then the correct way to handle the cube is as follows:

- If D is 12 pips or less, then the trailer must take.
- If D is 13 pips or more, then the trailer must pass.

To be as specific as possible, let's suppose that the leader's pipcount is 100 pips and the obtained LTP is 12 pips, then this means that the trailer must act as follows:

- If the trailer's pipcount is 112 pips or less, then the trailer must take.
- If the trailer's pipcount is 113 pips or more, then the trailer must pass.

The expression "*approach*" represents the ensemble of criteria, which allows you to decide whether you must double or not, redouble or not, take or pass.

1.2 Use of Snowie

Snowie can be used to evaluate the equity (or expected gain or expectation) of all analyzed positions. This software allows us to use the following types of evaluation:

- 1-Ply Fast
- 2-Ply Standard
- 3-Ply Precise
- Truncated Cubeless Rollout
- Full Cubeless Rollout
- Truncated Cubeful Rollout
- Full Cubeful Rollout
- Custom Rollout

The least precise type of evaluation is "1-Ply Fast". The "2-Ply Standard" type of evaluation is a little more precise, and the "3-Ply Precise" is again a little more precise.

The "Rollout" types of evaluation are more precise than the "3-Ply Precise" type of evaluation. Snowie allows us to make several types of rollout. The various types of rollout we can make with Snowie are the following ones: Truncated Cubeless, Full Cubeless, Truncated Cubeful and Full Cubeful. The "Cubeful" types of rollout are more precise than the "Cubeless" ones. The "Full" types of rollout are more precise than the "Truncated" ones. So, among these four types of rollout, the most precise one is obviously: "Full Cubeful".

Snowie also allows us to use the following type of evaluation: "Custom Rollout". This type of evaluation lets us choose any type of "Rollout" and it also allows us to specify the desired type of evaluation required to manipulate the checkers and the cube. The possible options are:

- 1-Ply Play
- 2-Ply Play
- 3-Ply Play
- 1-Ply Cube
- 2-Ply Cube
- 3-Ply Cube

The most precise type of evaluation we can make with Snowie therefore consists in using the "Custom Rollout" type of evaluation, with the following options: "Full Cubeful, 3-Ply Play, 3-Ply Cube".

When Snowie evaluates a "Money" position, the presented results always represent the equity of the position. Those results are presented either in the form of "Normalized points per game", or in the form of "Points per game". Snowie always specifies if his evaluation is presented in the form "Normalized points per game" or "Point per game". So we should always pay attention to this.

To transform the "Normalized points per game" in "Point per game", we simply have to multiply the obtained "Normalized points per game" by the level of the cube before it was handled.

For example, the result presented by Snowie could be: 0.500. This result corresponds to the evaluation of the equity as calculated by Snowie. This result could mean: 0.500 "Normalized points per game" or 0.500 "Points per game". With a bet of \$10.00/point, if the obtained result is 0.500 "Normalized points per game" and if the cube is in the center (meaning level 1), then Snowie's result means that the analyzed position offers an equity of \$5.00.

If the cube is at level 2 (no matter who has the cube in his possession), then Snowie's result of 0.500 "Normalized point per game" will mean that the analyzed position offers an equity of \$10.00.

If the cube is at level 4 (no matter who has the cube in his possession), then Snowie's result of 0.500 "Normalized points per game" will mean that the analysed position offers an equity of \$20.00.

We should never forget that a "Snowie's print position" is always expressed in "Normalized points per game". So when the cube is not in the center and when Snowie's equity is presented in the form of "Points per game", Snowie's results "on the screen" could be different than Snowie's results "on paper". To avoid any kind of confusion in this article, it is always specified if Snowie's result are expressed in "Normalized points per game" or in "Points per game".

Snowie also evaluates the precision of the equity obtained. The terms used to estimate the precision of an equity are the following ones: "95% confidence interval". This concept is a fundamental "statistics" concept. This last concept is mainly based on the centered and reduced normal law. The practical way to use the "95% confidence interval" concept is summarily explained below, but if you wish to get more theoretical information about this last concept, then you could consult statistics books, or you could also consult certain internet sites.

When Snowie gives an equity of 0.500 "Normalized points per game", with a "95% confidence interval" of 0.040 "Normalized points per game", this simply means that we are 95% certain that the true theoretical equity is situated between 0.460 and 0.540 "Normalized points per game". Consequently, this also means that 5% of the time, the true theoretical equity is situated below 0.460 "Normalized points per game" or over 0.540 "Normalized points per game". In this article, instead of using the expression "with a 95% confidence interval" of 0.040 "Normalized points per game", we could simply say that the precision of the result obtained is better than 0.040 "Normalized points per game". For example, instead of saying that a position was analyzed in order to obtain a "95% confidence interval" lower than 0.040 "Normalized points per game", we could simply say that the position was analyzed in order to obtain a precision better than 0.040 "Normalized points per game".

There is a relation between the following two elements:

- The precision of the obtained result.
- The number of played games.

So, the more precise results we wish to get, the more played games is necessary to get the desired precision.

There is a statistical law (or a mathematical formula) which allows us to conclude that to double a precision, we need four times more played games. For example, if to analyze a position to get a precision better than 0.040 "Normalized points per game" Snowie has to work 1 minute, then to get a precision twice as great, meaning to get a precision better than 0.020 "Normalized points per game", Snowie will have to work four times longer, meaning approximately 4 minutes. Consequently, to get a precision 4 times as great, meaning to get a precision better than 0.010 "Normalized points per game", Snowie will have to work 16 times longer, meaning approximately 16 minutes. And to obtain a precision 8 times greater, meaning to get a precision better than 0.005 "Normalized points per game", Snowie will have to work 64 times longer, meaning around 64 minutes.

The amount of time necessary for Snowie to make an analysis depends on several variables. One of the main ones is certainly the configuration of your computer. The amount of time necessary also depends on the position to be analyzed. For example, to analyze a running position where the pipcount of both players is about 100 pips, the amount of time necessary is approximately 8 times greater than the amount of time needed to analyze a running position where the pipcounts of both players is only about 50 pips.

Since the precision of a rollout depends essentially on the number of played games, to increase the precision of a rollout, it is necessary to increase the number of played games. Generally speaking, rollouts are relatively long to make, consequently, it is necessary to try to find a reasonable compromise between the desired precision and the period of time needed to make rollouts. Analyzing a position for personal purposes does not necessarily require a very high level of precision. For example 1,000 played games could be enough. But analyzing a position for publication purposes needs to be relatively more precise. For example, a minimum of 10,000 played games could be required. In the last case, it could be appropriate for the author (of any publication) to clearly indicate the precision of the obtained results. For example, to explain as clearly as possible what technique was used to evaluate the obtained results, the author could say something like this: "All the obtained equities evaluations have a precision better than 0.05 point per game". They could also say something like this: "All the obtained equities evaluations correspond to an equivalent of a minimum of 10,000 played games".

1.3 Use of Excel

Excel is a very well-known software program. It would not have been possible to elaborate on the theory presented in this article without the use of this software. Excel was mainly used for:

- Making complicated and repetitive calculations.
- Presenting the obtained results in the form of tables and graphs.

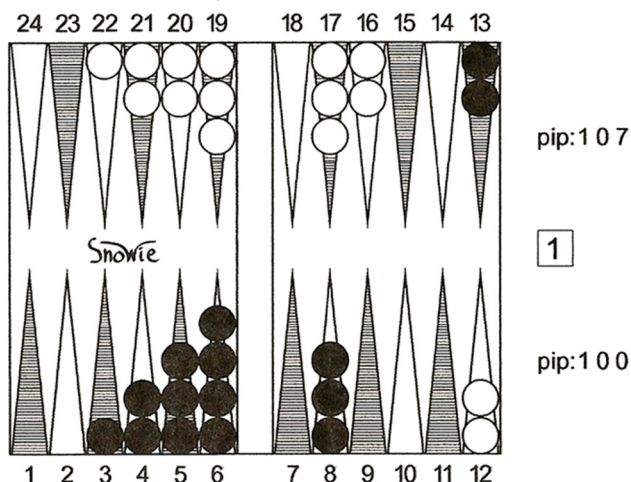
1.4 Obtained results for a leader's pipcount of 100 pips

In this section, we will develop in great detail, the technique used to calculate DP (Doubling Point), RP (Redoubling Point), and LTP (Last Take Point) for a leader's pipcount of 100 pips. To perfectly understand this section, you necessarily have to fully understand all the concepts previously elaborated.

To get the desired results, it is necessary to use 2 reference positions, namely one position with the cube in the center and a second position which corresponds exactly to the first position, but where the cube belongs to Black.

The first reference position is as follows:

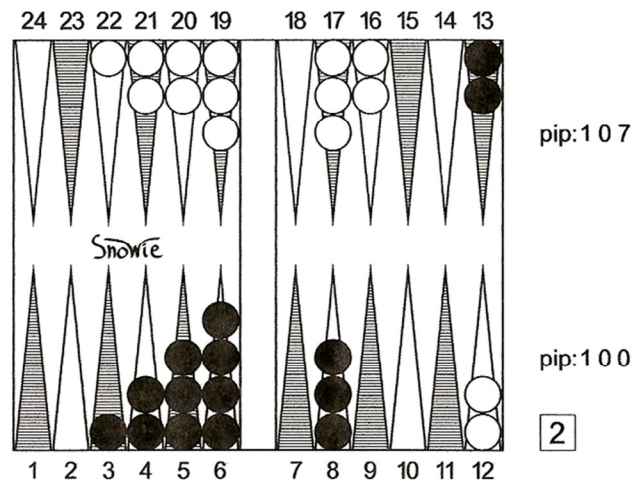
Position No. 1
(Cube in the center)



Position no. 1 allows us to calculate DP and LTP.

The second reference position is as follows:

Position No. 2
(Cube to Black)



Position no. 2 allows us to calculate RP and LTP.

In these two reference positions, the pipcounts are Black = 100 pips, White = 107 pips and it is Black to play. Therefore in these two positions, Black is the leader, "P" (the leader's Pipcount) is 100 pips, and "A" (the leader's Advantage) is 7 pips.

In order to calculate DP, RP and LTP, it is necessary to analyze several other positions. We will analyze all the positions in which "A" varies from 5 pips to 15 pips. To obtain all the other positions, we only have to take the checker located on the 22-point and change its place. For example, to obtain the position in which "A" is 6 pips, we simply have to remove the checker from the 22-point and place it on the 23-point. Indeed, with this change, White's pipcount is 106 pips and "A" is 6 pips. To obtain the position in which "A" is 8 pips, we simply have to remove the checker from the 22-point and place it on the 21-point. By moving the checker from the 22-point to some another points, it is possible to get all other positions in which "A" can varies from 5 pips to 15 pips.

To analyze the position where the leader's pipcount is 100 pips, it is necessary to use Snowie and have it analyze all the positions where "A" varies from 5 pips to 15 pips. To make this analysis, the type of evaluation used is "3-Ply Precise". This type of evaluation allows any "skeptical" reader to verify all the results and calculations which will be made soon. We didn't use a rollout type of evaluation, because it would not have allowed a skeptical reader to really verify all the calculations presented below.

Table 1 presents the obtained equities when the cube is in the center.

Table 2 presents the obtained equities when the cube belongs to Black.

Based on table 1, graph 1 illustrates the three following curves:

- "No Double, Take" curve
- "Double, Take" curve
- "Double, Pass" curve

Graph 1 presents and introduces two new concepts, namely the DPth and the LTPth. DPth is the abbreviation of "theoretical Doubling Point". LTPth is the abbreviation of "theoretical Last Take Point". The value of the DPth is obtained when the "No Double, Take" curve meets the "Double, Take" curve. The value of the LTPth is obtained when the "Double, Take" curve meets the "Double, Pass" curve.

Based on table 2, graph 2 illustrates the three following curves:

- "No Redouble, Take" curve
- "Redouble, Take" curve
- "Redouble, Pass" curve

Graph 2 presents and introduces one more new concept, namely the RPth. RPth is the abbreviation of "theoretical Redoubling Point". The value of the RPth is obtained when the "No Redouble, Take" curve meets the "Redouble, Take" curve. Graph 2 also illustrates the LTPth. This point is obtained when the "Redouble, Take" curve meets the "Redouble, Pass" curve.

Graphs 1 and 2 show that:

- 1) DPth, as illustrated on graph 1, is the meeting point between the "No Double, Take" curve and the "Double, Take" curve. This point is located near 7.9 pips.
- 2) RPth, as illustrated on graph 2, is the meeting point between the "No Redouble, Take" curve and the "Redouble, Take" curve. This point is located near 9.8 pips.
- 3) LTPth, as illustrated on graph 1, is the meeting point between the "Double, Take" curve and the "Double, Pass" curve. This point is located near 12.8 pips.
- 4) LTPth, as illustrated on graph 2, is the meeting point between the "Redouble, Take" curve and the "Redouble, Pass" curve. This point is located near 12.8 pips. Please note that the value of the LTPth on graph 1 is identical to the value of the LTPth on graph 2.

So, with graphs 1 and 2, it is possible to visually estimate the values of the DPth, the RPth and the LTPth. This evaluation is a visual one; consequently it is not very precise. To obtain a more precise evaluation, it is necessary to use a mathematical technique and not just a visual one.

To calculate the DPth with the use of a mathematical approach, it is not necessary to use all the data of the "No Double, Take" curve and of the "Double, Take" curve. In fact, we only have to use the data relating to 2 positions, namely the one which is immediately below the DPth and the one which is immediately above the DPth. Since the visual DPth evaluation is: near 7, 9 pips; the data needed to calculate DPth are $A = 7$ pips and $A = 8$ pips. Appendix 1 explains in detail the mathematical technique to be used. The result obtained is $DPth = 7.90$ pips. This result is presented with 2 decimals (and not just with one). This indicates that the second decimal is significant.

To calculate the RPth with the use of a mathematical approach, it is necessary to use the exact same technique as the one used to calculate the DPth. Since the visual RPth evaluation is: near 9, 8 pips; the data needed to calculate RPth are $A = 9$ pips and $A = 10$ pips. Appendix 2 explains in detail the mathematical technique to be used. The result obtained is: $RPth = 9.80$ pips.

To calculate the LTPth with the use of a mathematical approach, it is also necessary to use the exact same technique as the one used to calculate the DPth or the RPth. The LTPth can be calculated by either using the data of table 1 or that of table 2. Whatever data is used, the result obtained will be exactly the same. Since the visual LTPth evaluation is: near 12, 8 pips; the data needed to calculate LTPth are $A = 12$ pips and $A = 13$ pips. Appendix 3 explains in detail the mathematical technique to be used. The result obtained is $LTPth = 12.81$ pips.

In summary, by using the "3-Ply Precise" type of evaluation, the obtained results are:

Criterion	3-Ply Precise
DPth	7.90 pips
RPth	9.80 pips
LTPth	12.81 pips

If the analyzed positions were different and if the type of evaluation used was different, then the obtained results could possibly be different, but you can be assured that the obtained results would be very similar.

1.5 Obtained results for a leader's pipcount ranging from 20 pips to 120 pips

The technique used to calculate the DPth, the RPth and the LTPth when the leader pipcount is 100 pips was previously elaborated in detail.

The DPth, the RPth and the LTPth for all the leader's pipcounts ranging from 20 pips to 120 pips, with an interval of 2 pips, were calculated by using the exact same technique as the one previously explained. 51 positions were analyzed, namely: 20 pips, 22 pips, 24 pips..., 116 pips, 118 pips and 120 pips. The type of evaluation used is: "Full Cubeful Rollout, 3-Ply Play, 3-Ply Cube" in order to obtain a precision better than 0.040 "Normalized points per games ". The positions used are "low-wastage positions" which meet the criteria enumerated in section 1.1.3. For each of these 51 pipcounts, 3 results were obtained, namely the DPth, the RPth and the LTPth. The 153 obtained results are presented in table 3. These results are also illustrated in graph 3.

In order to obtain the 153 results in question with the required precision, it was necessary to perform a relatively significant amount of work which took quite a while to do. Among others, Snowie had "to work" for several hundred hours. EXCEL was used to make the necessary interpolation calculations and to present the obtained results in the form of tables and graphs.

If you ever decide to perform the exact same exercise, your results could obviously be very slightly different, because your positions would necessarily be different from those that were in fact used to carry out this analysis. But, you can be sure that all the results you get will be very similar to those illustrated in graph 3.

1.6 Theoretical curves of the optimal approach

Graph 4 illustrates the theoretical curves of the optimal approach. In fact, this graph is a combination of the following three curves: DPth, RPth and LTPth of the optimal approach. To obtain these curves it is necessary to proceed as follows:

- Right-click on one of the dots illustrated on graph 3 and a popup menu appears.
- Click on : "Add trendline... "
- Choose the curve: "Polynomial"
- Choose "2" as the "Order" of this curve.

Graph 5 illustrates the three curves so obtained. Those curves represent the theoretical curves of the optimal approach. In fact, these curves produce the DPth values, the RPth values and the LTPth values of the optimal approach.

Table 4 presents the obtained DPth, RPth and LTPth values of the optimal approach. Those values correspond to the values as illustrated on graph 5.

Graph 6 illustrates the DPth curve and the DP curve of the optimal approach for a leader's pipcount ranging from 20 pips to 120 pips. In order to obtain a DP value, it is necessary to "round up" the DPth values. Consequently, you could observe that the DP curve is above the DPth curve.

Graph 7 illustrates the RPth curve and the RP curve of the optimal approach for a leader's pipcount ranging from 20 pips to 120 pips. In order to obtain a RP value, it is necessary to "round up" the RPth values. Consequently, you could observe that the RP curve is above the RPth curve.

Graph 8 illustrates the LTPth curve and the LTP curve of the optimal approach for a leader's pipcount ranging from 20 pips to pips 120. In order to obtain a LTP value, it is necessary to "round down" the LTPth values. Consequently, you could observe that the LTP curve is below the LTPth curve. .

Graph 9 illustrates a combination of graph 6, graph 7 and graph 8.

Table 5 presents the obtained DP, the RP and the LTP values for a leader's pipcount ranging from 20 pips to 120 pips.

1.7 Table of marginal decision points

To present the optimal approach, it is possible to use several different techniques. For example, we have already seen that the DP, RP and LTP values could be obtained by using one of the following techniques:

- Using several graphs, such as graph 6, 7 and 8.
- Using table 5.

It is also possible to use the concept of "marginal decision points". A "marginal decision point" is defined as a meeting point between a practical value curve like the DP curve, the RP curve or the LTP curve with a theoretical value curve like the DPth curve, the RPth curve or the LTPth curve. In graphs 6, 7 and 8, the marginal decision points are highlighted.

Table 6 presents the marginal decision points of the optimal approach. The abbreviation "MDP" means "Marginal Decision Point". Table 6 shows the exact same information as presented in in graphs 6, 7, 8 or in table 5.

Here is the way to correctly use any table of marginal decision points like table 6:

To decide if the leader must double, use MDP which is equal or immediately above the leader's pipcount; the corresponding DP result is the minimum advantage required to double.

To decide if the leader must redouble, use MDP which is equal or immediately above the leader's pipcount; the corresponding RP result is the minimum advantage required to redouble.

To decide if the trailer must take, use MDP which is equal or immediately below the leader's pipcount; the corresponding LTP result is the maximum disadvantage required to take.

To help you to use table 6, you could use the following "mnemonic tips":

- 1) When you are the leader and you have to decide whether to double or redouble, you do it in order to increase your gain. So you must use the leader's pipcount upwards. For example, if your pipcount is 100 pips, you should use the MDP value of 105 pips; the corresponding DP is therefore 8 pips. And to redouble, you should use the MDP value of 107 pips; the corresponding RP is therefore 10 pips.
- 2) When you are the trailer and you must decide whether to take or pass, you do it in order to minimize your losses. So you must use the leader's opponent's pipcount downwards. For example, if the leader's pipcount is 100 pips, you should use the MDP value of 96 pips; the corresponding LTP value is therefore 12 pips.

1.8 Marginal decision points curves

It is also possible to present an approach by using the concept of "marginal decision points curves".

Graph 9 illustrates the marginal decision points curves of the optimal approach. This graph illustrates a combination of the three following curves: the DP marginal decision points curve, the RP marginal decision points curve and the LTP marginal decision points curve. To obtain these curves, the data presented in table 6 must be taken into account and we should use the same technique as already explained in section 1.6.

Graph 10 illustrates the three following curves:

- DP marginal decision points curve
- RP marginal decision points curve
- LTP marginal decision points curve

1.9 Different possible techniques to obtain DP, RP and LTP

The different techniques that you can use to obtain DP, RP and LTP are the following one:

- Using a graph like graphs 6, 7 and 8.
- Using a table like table 5.
- Using a table of marginal decision points like table 6

Whatever technique is used to present the optimal approach, the obtained result will always be exactly the same.

For example, if "P" (the leader's pipcount) is 100 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 108 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 110 pips.
- The trailer must take if his pipcount is equal or inferior to 112 pips.

For example, if "P" (the leader's pipcount) is 50 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 53 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 54 pips.
- The trailer must take if his pipcount is equal or inferior to 56 pips.

1.10 The LTP marginal decision points curve as obtained by Trice

Walter Trice has also worked to develop a LTP marginal decision points curve. Mr. Trice presented the results he obtained in the book: "Backgammon BOOT CAMP" in the chapter: "From Pipcount to Cube Action". He does not explain the technique he used to obtain his results, but he presents a table which allows us to deduct the LTP marginal decision points curve. The results he obtained are presented as follows:

Leader's Pipcount	Trailer can take, if down:
19 – 25	2
26 – 32	3
33 – 39	4
40 – 46	5
47 – 53	6
54 – 61	7
62 – 69	8
70 – 78	9
79 – 88	10
89 – 99	11
100 – 110	12
111 – 122	13

Using the concept of Marginal Decision Points, the LTP values as obtained by Trice are:

MDP	LTP
19	2
26	3
33	4
40	5
47	6
54	7
62	8
70	9
79	10
89	11
100	12
111	13
123	14

Using this approach, when "P" (the leader's pipcount) is 100 pips, the trailer must take if his pipcount is equal or inferior to 112 pips.

Using this approach, when "P" (the leader's pipcount) is 50 pips, the trailer must take if his pipcount is equal or inferior to 56 pips.

Graph 11 illustrates the LTP values as obtained by Trice.

Graph 12 illustrates the LTP marginal decision points curve as obtained by Trice. Please note that together, the "theoretical" LTP marginal decision points curve as obtained by Trice is a curve (and not a "straight line").

Graph 13 illustrates the 2 following curves:

- The LTP marginal decision points curve as obtained by Trice (in RED).
- The LTP marginal decision points curve as obtained by the optimal approach (in BLACK).

It is interesting to observe that those two curves are practically identical.

The fact that the differences are very small is probably connected to the fact that in order to develop his theoretical approach to obtain the LTP marginal decision points, Mr. Trice probably used a calculation technique pretty similar to the one presented in this section.

Mr. Trice has probably done similar calculations relating to the other values such as the RP and the DP values, but unfortunately he did not present any of the obtained results.

1.11 Tables, graphs and appendices

Table 1:	Obtained equities for a leader's pipcount of 100 pips when the cube is in the center
Table 2:	Obtained equities for a leader's pipcount of 100 pips when the cube belongs to Black
Table 3:	Obtained DPth, RPth and LTPth values for a leader's pipcount ranging from 20 pips to 120 pips
Table 4:	Obtained DPth, RPth and LTPth values of the optimal approach
Table 5:	Obtained DP, RP and LTP values for the optimal approach
Table 6:	Table of marginal decision points of the optimal approach
Graph 1:	Obtained equities for a leader's pipcount of 100 pips when the cube is in the center
Graph 2:	Obtained equities for a leader's pipcount of 100 pips when the cube belongs to Black
Graph 3:	Obtained DPth, RPth and LTPth values for a leader's pipcount ranging from 20 pips to 120 pips
Graph 4:	Theoretical curves of the optimal approach
Graph 5:	Obtained DPth, RPth and LTPth values of the optimal approach
Graph 6:	DP curve of the optimal approach
Graph 7:	RP curve of the optimal approach
Graph 8:	LTP curve of the optimal approach
Graph 9:	Marginal decision points curves of the optimal approach
Graph 10:	DP, RP and LTP marginal decision points curves of the optimal approach
Graph 11:	LTP values as obtained by Trice
Graph 12:	LTP marginal decision points curve as obtained by Trice
Graph 13:	LTP marginal decision points curve as obtained by Trice Vs the optimal approach
Appendix 1:	Calculation of the DPth
Appendix 2:	Calculation of the RPth
Appendix 3:	Calculation of the LTPth

**Table 1: Obtained equities for a leader's pipcount
of 100 pips when the cube is in the center**

Leader's advantage	Equity No Double, Take	Equity Double, Take	Equity Double, Pass	Proper Cube Action
5	0.512	0.451	1.000	No Double, Take
6	0.560	0.523	1.000	No Double, Take
7	0.614	0.596	1.000	No Double, Take
8	0.667	0.669	1.000	Double, Take
9	0.703	0.719	1.000	Double, Take
10	0.754	0.792	1.000	Double, Take
11	0.802	0.868	1.000	Double, Take
12	0.851	0.940	1.000	Double, Take
13	0.886	1.014	1.000	Double, Pass
14	0.903	1.056	1.000	Double, Pass
15	0.923	1.109	1.000	Double, Pass

Note: All equities presented in table 1 are expressed in "Point per game".

**Table 2: Obtained equities for a leader's pipcount of 100 pips
when the cube belongs to Black**

Leader's advantage	Equity No Redouble, Take	Equity Redouble, Take	Equity Redouble, Pass	Proper Cube Action
5	1.186	0.902	2.000	No Redouble, Take
6	1.260	1.046	2.000	No Redouble, Take
7	1.346	1.192	2.000	No Redouble, Take
8	1.430	1.338	2.000	No Redouble, Take
9	1.486	1.438	2.000	No Redouble, Take
10	1.572	1.584	2.000	Redouble, Take
11	1.650	1.736	2.000	Redouble, Take
12	1.734	1.880	2.000	Redouble, Take
13	1.798	2.028	2.000	Redouble, Pass
14	1.828	2.112	2.000	Redouble, Pass
15	1.864	2.218	2.000	Redouble, Pass

Note: All equities presented in table 2 are expressed in "Point per game"

Table 3: Obtained DPth, RPth and LTPth values for a leader's pipcount ranging from 20 pips to 120 pips

P	DPth	RPth	UTPth
20	-1,43	-0,84	2,29
22	-1,21	-0,80	1,82
24	-0,16	-0,07	3,16
26	-0,31	0,38	3,23
28	-1,26	0,21	3,42
30	-0,11	0,67	4,19
32	0,09	0,77	4,05
34	0,56	1,35	4,70
36	0,45	0,95	5,14
38	0,21	1,25	4,84
40	1,37	2,17	5,16
42	1,91	2,52	4,87
44	1,88	2,67	6,06
46	2,03	3,56	5,76
48	2,44	3,10	6,30
50	2,47	4,21	7,18
52	2,63	3,35	7,33
54	2,35	3,40	7,19
56	3,34	4,00	8,14
58	3,16	5,43	8,38
60	3,75	5,40	8,12
62	4,43	5,79	8,63
64	4,24	5,51	8,69
66	4,02	5,78	9,17
68	4,60	6,33	8,81
70	4,86	6,22	8,75
72	5,19	6,44	9,13
74	4,52	6,37	9,62
76	4,85	7,65	9,40
78	6,00	7,46	9,79
80	5,33	7,04	10,09
82	6,50	7,90	10,33
84	5,48	8,07	10,79
86	5,79	8,61	10,62
88	5,79	8,04	11,26
90	6,75	8,50	11,11
92	7,13	7,70	12,11
94	6,42	8,82	11,53
96	6,32	9,21	12,09
98	6,88	8,11	11,54
100	7,30	9,37	12,10
102	7,57	9,29	13,59
104	7,89	9,85	13,09
106	8,45	10,05	12,35
108	8,46	10,21	12,46
110	8,40	10,70	13,56
112	8,05	10,23	13,44
114	9,17	10,22	14,21
116	9,45	10,33	14,03
118	8,68	10,43	14,01
120	9,68	11,59	15,06

Table 4: Obtained DPth, RPth and LTPth values of the optimal approach

P	DPth	RPth	LTPth
20	-1,32	-1,00	2,30
21	-1,19	-0,84	2,46
22	-1,06	-0,68	2,62
23	-0,92	-0,52	2,78
24	-0,79	-0,36	2,94
25	-0,66	-0,20	3,10
26	-0,53	-0,04	3,26
27	-0,40	0,12	3,42
28	-0,26	0,28	3,58
29	-0,13	0,44	3,74
30	0,00	0,60	3,90
31	0,12	0,76	4,04
32	0,24	0,92	4,18
33	0,36	1,08	4,32
34	0,48	1,24	4,46
35	0,60	1,40	4,60
36	0,72	1,56	4,74
37	0,84	1,72	4,88
38	0,96	1,88	5,02
39	1,08	2,04	5,16
40	1,20	2,20	5,30
41	1,32	2,34	5,44
42	1,44	2,48	5,58
43	1,56	2,62	5,72
44	1,68	2,76	5,86
45	1,80	2,90	6,00
46	1,92	3,04	6,14
47	2,04	3,18	6,28
48	2,16	3,32	6,42
49	2,28	3,46	6,56
50	2,40	3,60	6,70
51	2,51	3,74	6,83
52	2,62	3,88	6,96
53	2,73	4,02	7,09
54	2,84	4,16	7,22
55	2,95	4,30	7,35
56	3,06	4,44	7,48
57	3,17	4,58	7,61
58	3,28	4,72	7,71
59	3,39	4,86	7,83
60	3,50	5,00	7,95
61	3,61	5,13	8,08
62	3,72	5,25	8,21
63	3,83	5,38	8,34
64	3,94	5,50	8,48
65	4,05	5,63	8,60
66	4,16	5,75	8,72
67	4,27	5,88	8,84
68	4,38	6,00	8,96
69	4,49	6,13	9,08
70	4,60	6,25	9,20

P	DPth	RPth	LTPth
71	4,70	6,37	9,31
72	4,80	6,48	9,42
73	4,90	6,60	9,53
74	5,00	6,71	9,64
75	5,10	6,83	9,75
76	5,20	6,94	9,86
77	5,30	7,06	9,97
78	5,40	7,17	10,08
79	5,50	7,29	10,19
80	5,60	7,40	10,30
81	5,70	7,50	10,41
82	5,80	7,60	10,52
83	5,90	7,70	10,63
84	6,00	7,80	10,74
85	6,10	7,90	10,85
86	6,20	8,00	10,96
87	6,30	8,10	11,07
88	6,40	8,20	11,18
89	6,50	8,30	11,29
90	6,60	8,40	11,40
91	6,69	8,49	11,50
92	6,78	8,58	11,60
93	6,87	8,67	11,70
94	6,96	8,76	11,80
95	7,05	8,85	11,90
96	7,14	8,94	12,00
97	7,23	9,03	12,10
98	7,32	9,12	12,20
99	7,41	9,21	12,30
100	7,50	9,30	12,40
101	7,59	9,39	12,50
102	7,68	9,48	12,60
103	7,77	9,57	12,70
104	7,86	9,66	12,80
105	7,95	9,75	12,90
106	8,04	9,84	13,00
107	8,13	9,93	13,10
108	8,22	10,02	13,20
109	8,31	10,11	13,30
110	8,40	10,20	13,40
111	8,48	10,27	13,49
112	8,56	10,34	13,57
113	8,64	10,41	13,66
114	8,72	10,48	13,74
115	8,80	10,55	13,83
116	8,88	10,62	13,91
117	8,96	10,69	14,00
118	9,04	10,76	14,08
119	9,12	10,83	14,17
120	9,20	10,90	14,25

Table 5: Obtained DP, RP and LTP values for the optimal approach

P	DP	RP	LTP
20	-1	-1	2
21	-1	0	2
22	-1	0	2
23	0	0	2
24	0	0	2
25	0	0	3
26	0	0	3
27	0	1	3
28	0	1	3
29	0	1	3
30	0	1	3
31	1	1	4
32	1	1	4
33	1	2	4
34	1	2	4
35	1	2	4
36	1	2	4
37	1	2	4
38	1	2	5
39	2	3	5
40	2	3	5
41	2	3	5
42	2	3	5
43	2	3	5
44	2	3	5
45	2	3	6
46	2	4	6
47	3	4	6
48	3	4	6
49	3	4	6
50	3	4	6
51	3	4	6
52	3	4	6
53	3	5	7
54	3	5	7
55	3	5	7
56	4	5	7
57	4	5	7
58	4	5	7
59	4	5	7
60	4	5	7
61	4	6	8
62	4	6	8
63	4	6	8
64	4	6	8
65	5	6	8
66	5	6	8
67	5	6	8
68	5	6	8
69	5	7	9
70	5	7	9

P	DP	RP	LTP
71	5	7	9
72	5	7	9
73	5	7	9
74	6	7	9
75	6	7	9
76	6	7	9
77	6	8	9
78	6	8	10
79	6	8	10
80	6	8	10
81	6	8	10
82	6	8	10
83	6	8	10
84	6	8	10
85	7	8	10
86	7	8	10
87	7	9	11
88	7	9	11
89	7	9	11
90	7	9	11
91	7	9	11
92	7	9	11
93	7	9	11
94	7	9	11
95	8	9	11
96	8	9	12
97	8	10	12
98	8	10	12
99	8	10	12
100	8	10	12
101	8	10	12
102	8	10	12
103	8	10	12
104	8	10	12
105	8	10	12
106	9	10	13
107	9	10	13
108	9	11	13
109	9	11	13
110	9	11	13
111	9	11	13
112	9	11	13
113	9	11	13
114	9	11	13
115	9	11	13
116	9	11	13
117	9	11	14
118	10	11	14
119	10	11	14
120	10	11	14

Table 6: Table of marginal decision points of the optimal approach

MDP	DP
22	-1
30	0
38	1
46	2
55	3
64	4
73	5
84	6
94	7
105	8
117	9

MDP	RP
20	-1
26	0
32	1
38	2
45	3
52	4
60	5
68	6
76	7
86	8
96	9
107	10
120	11

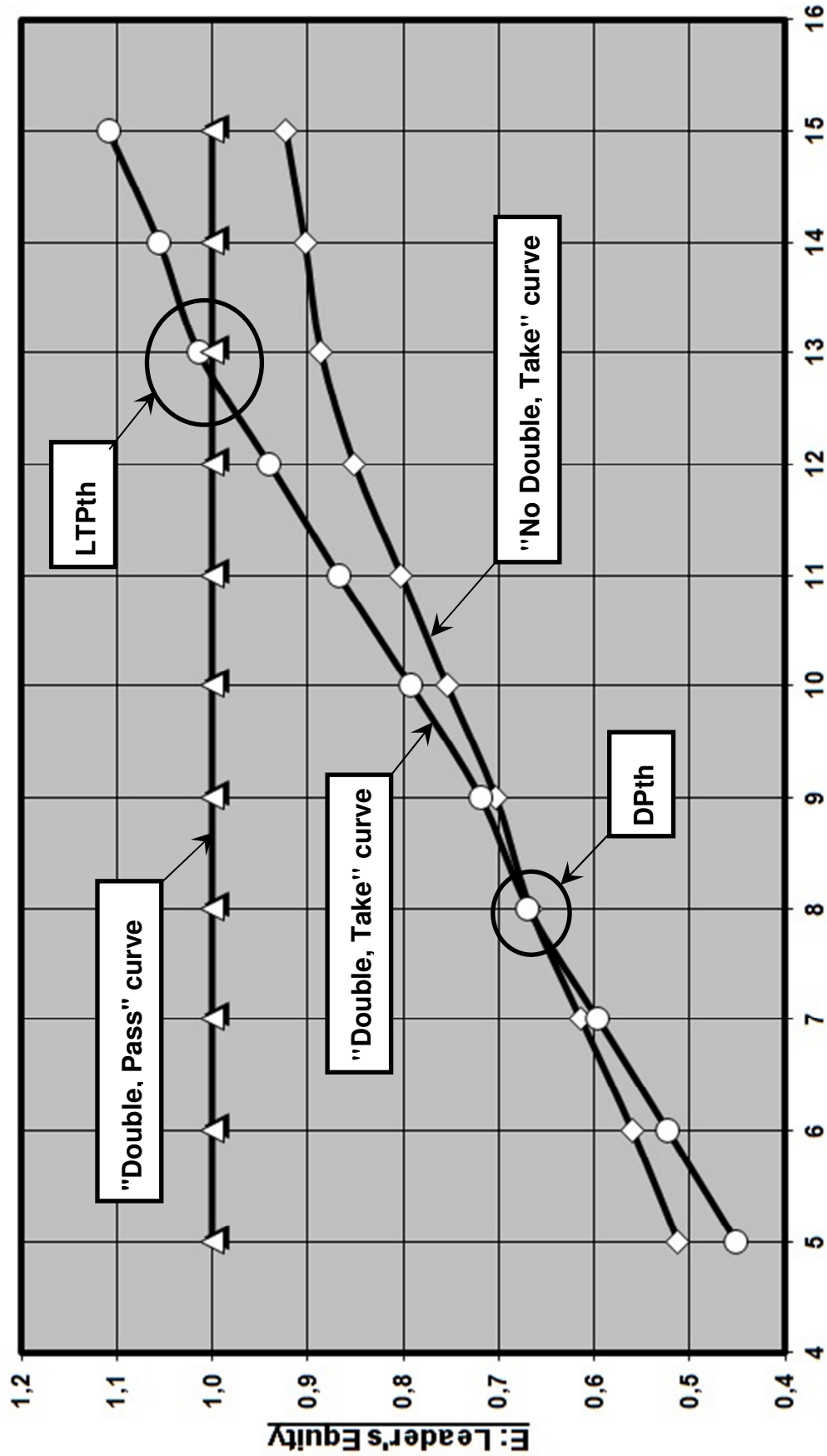
MDP	LTP
25	3
31	4
38	5
45	6
53	7
61	8
69	9
78	10
87	11
96	12
106	13
117	14

To decide if the leader must double, use MDP which is equal or immediately above the leader's pipcount; the corresponding DP result is the minimum advantage required to double.

To decide if the leader must redouble, use MDP which is equal or immediately above the leader's pipcount; the corresponding RP result is the minimum advantage required to redouble.

To decide if the trailer must take, use MDP which is equal or immediately below the leader's pipcount; the corresponding LTP result is the maximum disadvantage required to take.

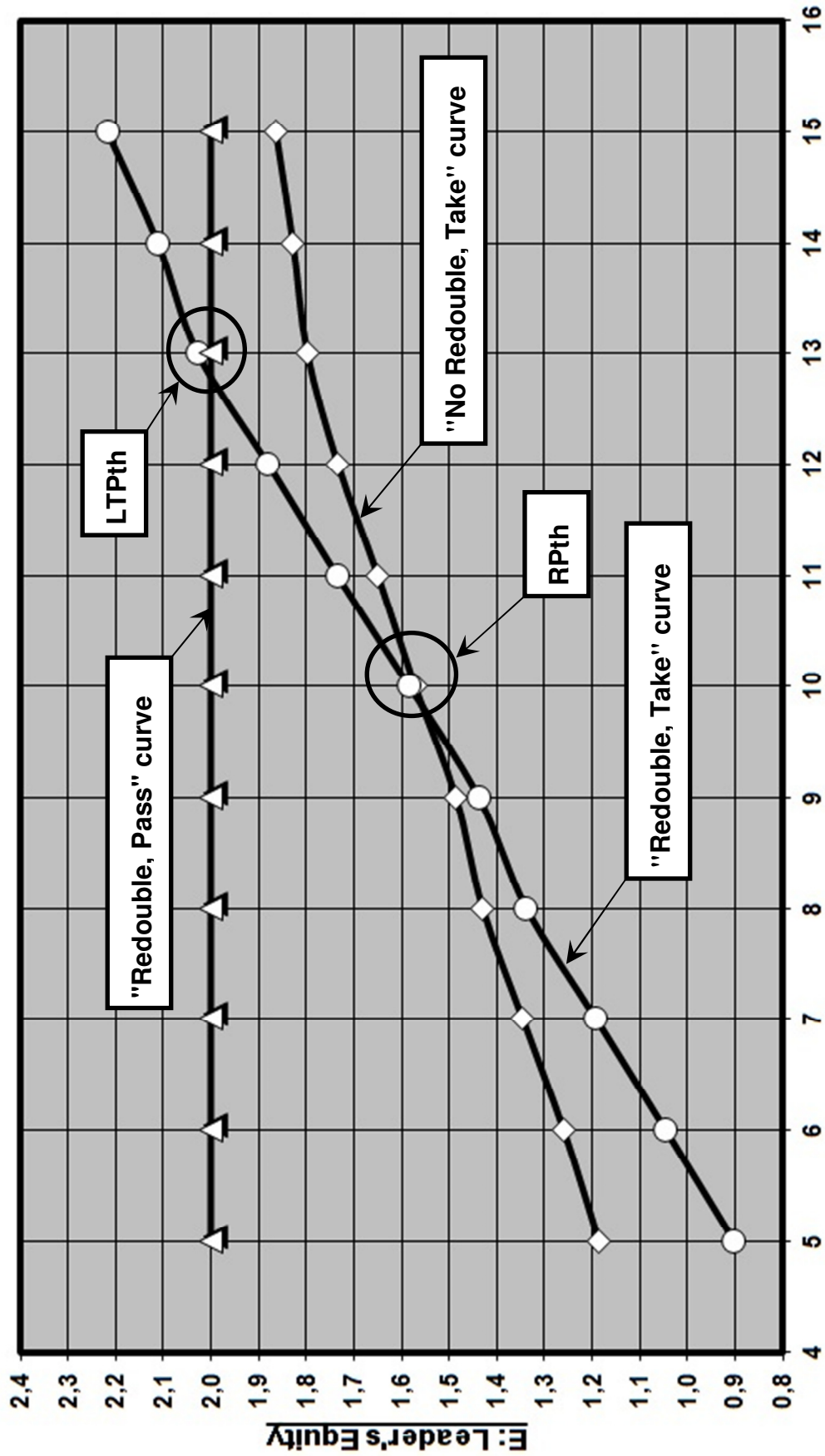
**Graph 1: Obtained equities for a leader's pipcount of 100 pips
when the cube is in the center**



A: Leader's Advantage

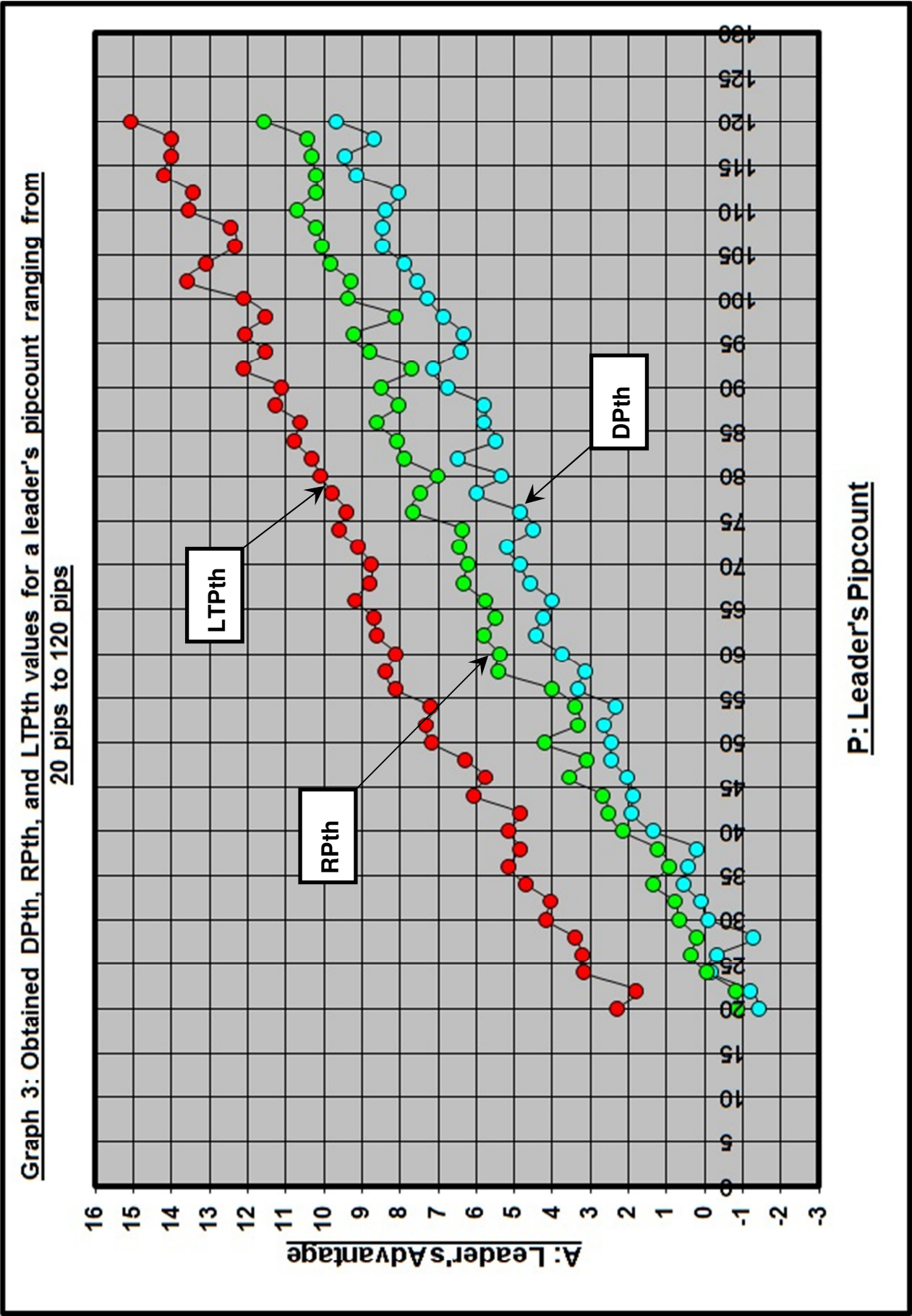
E: Leader's Equity

**Graph 2: Obtained equities for a leader's pipcount of 100 pips
when the cube belongs to Black**

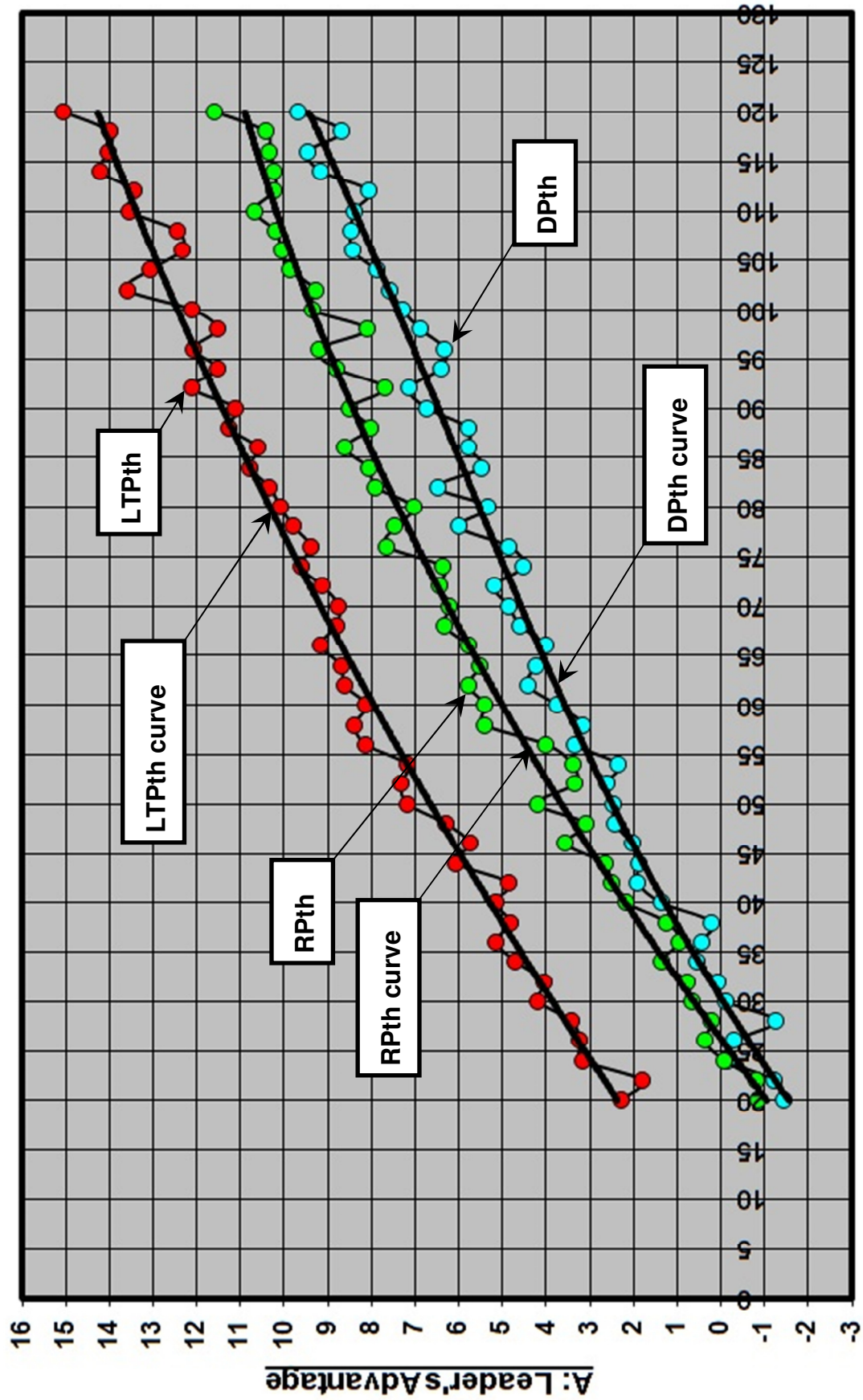


A: Leader's Advantage

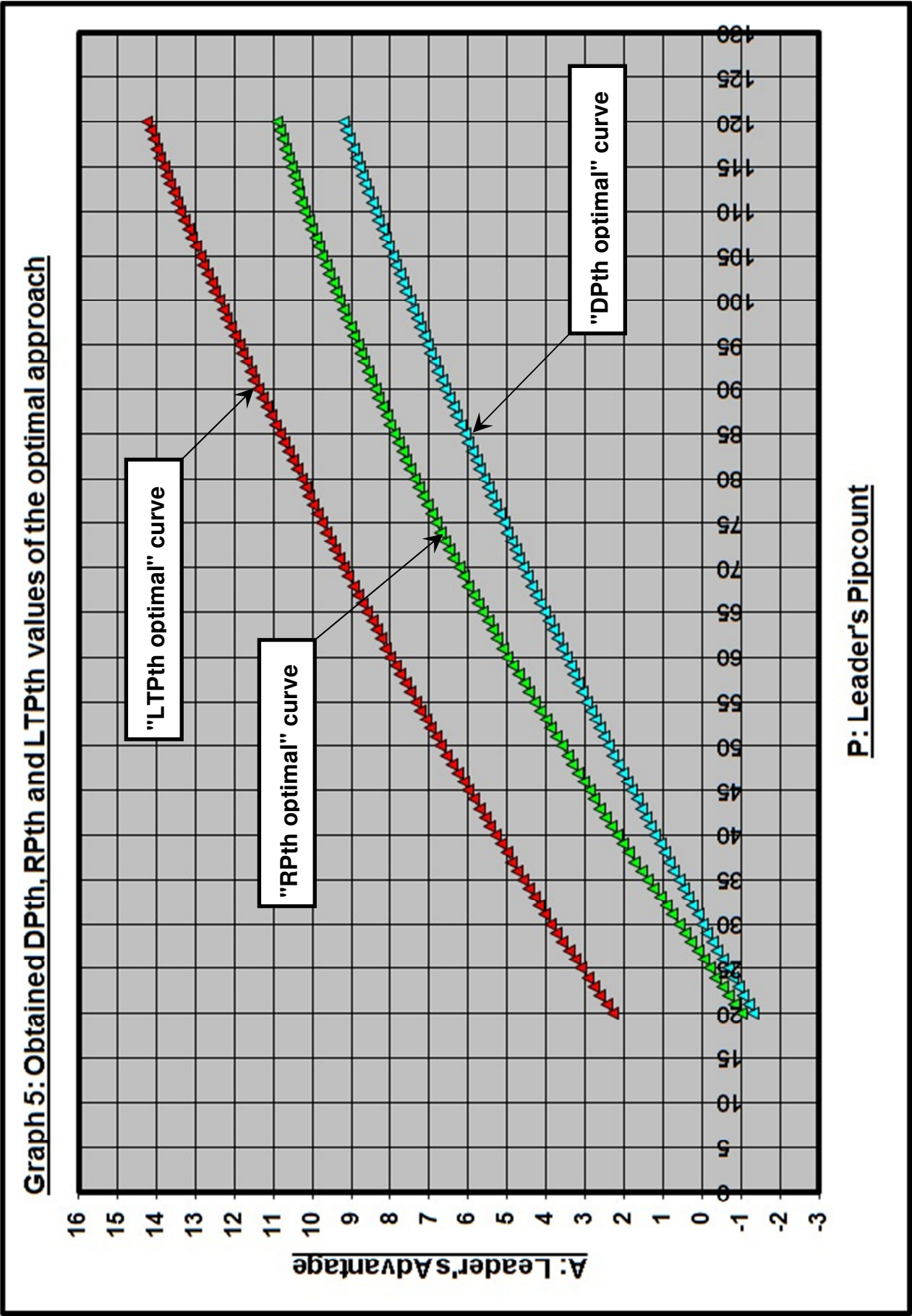
E: Leader's Equity



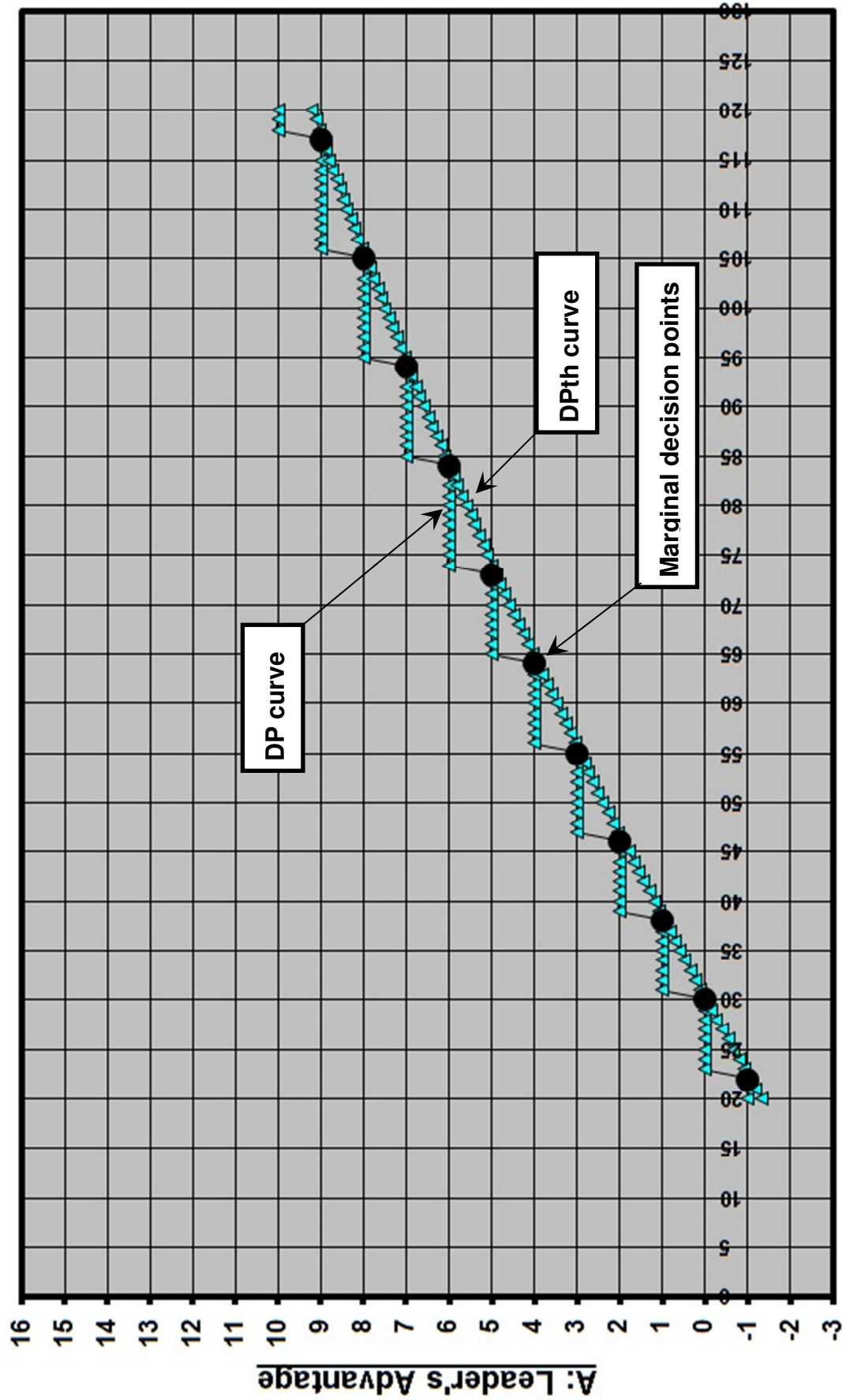
Graph 4: Theoretical curves of the optimal approach



P: Leader's Pipcount

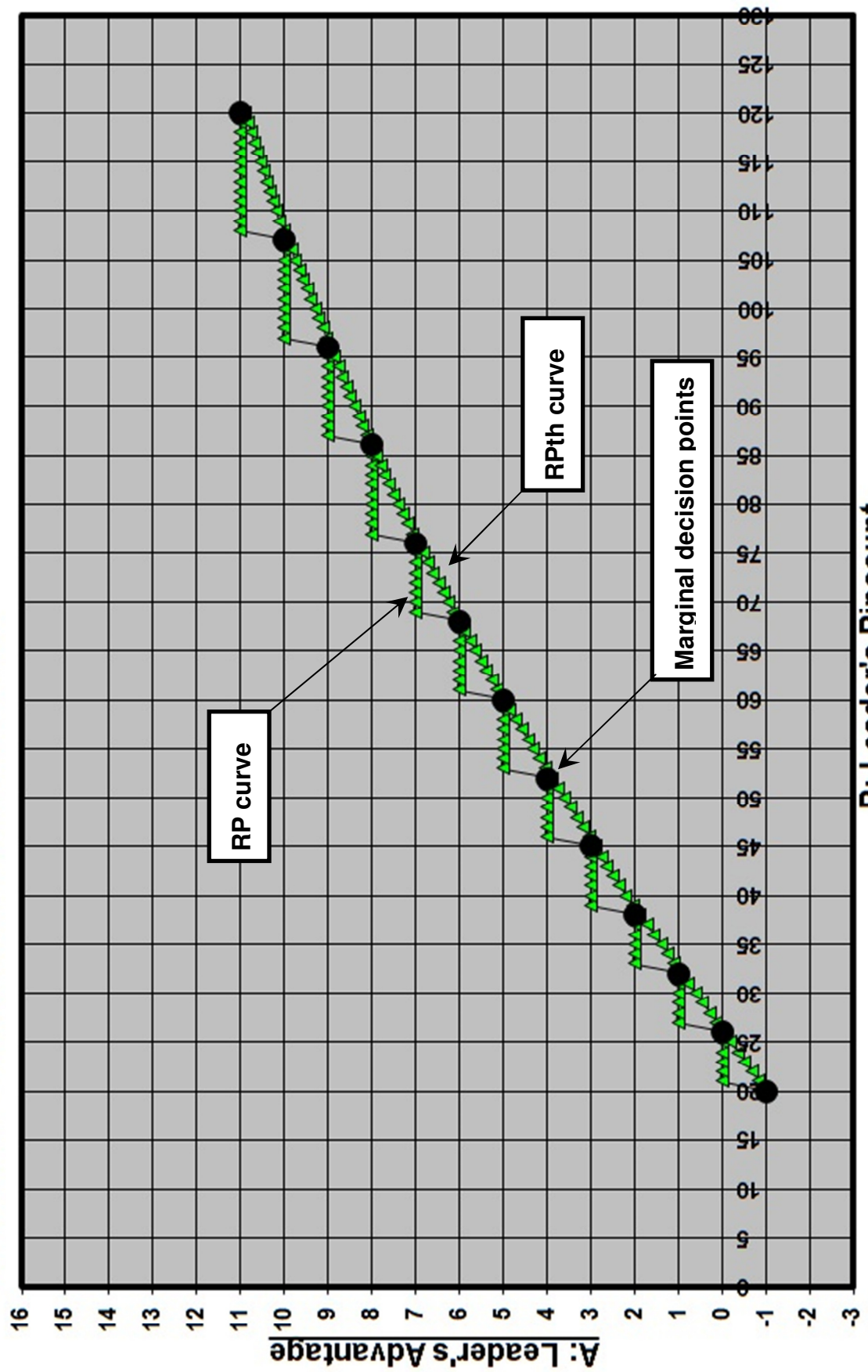


Graph 6: DP curve of the optimal approach



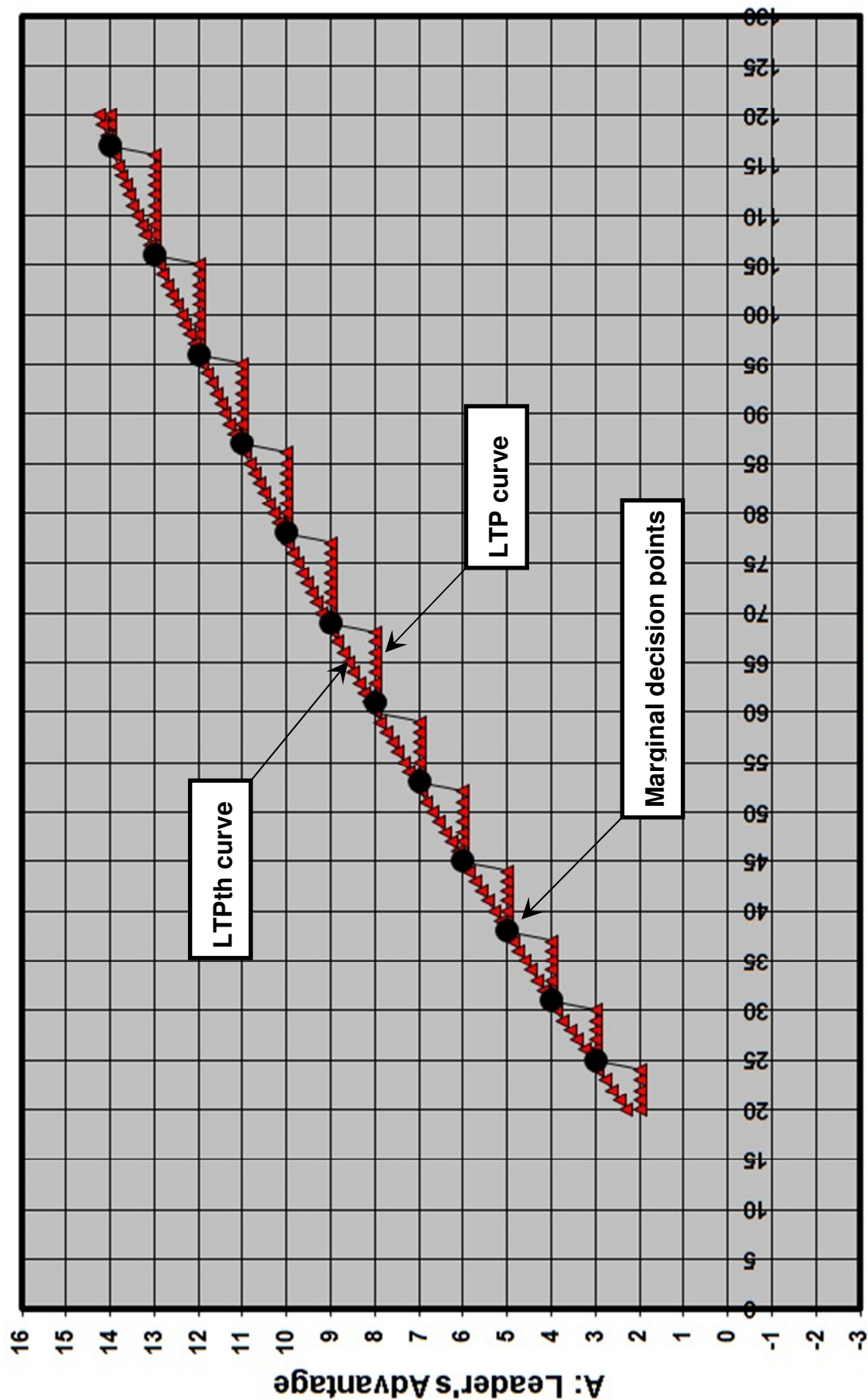
P: Leader's Pipcount

Graph 7: RP curve of the optimal approach

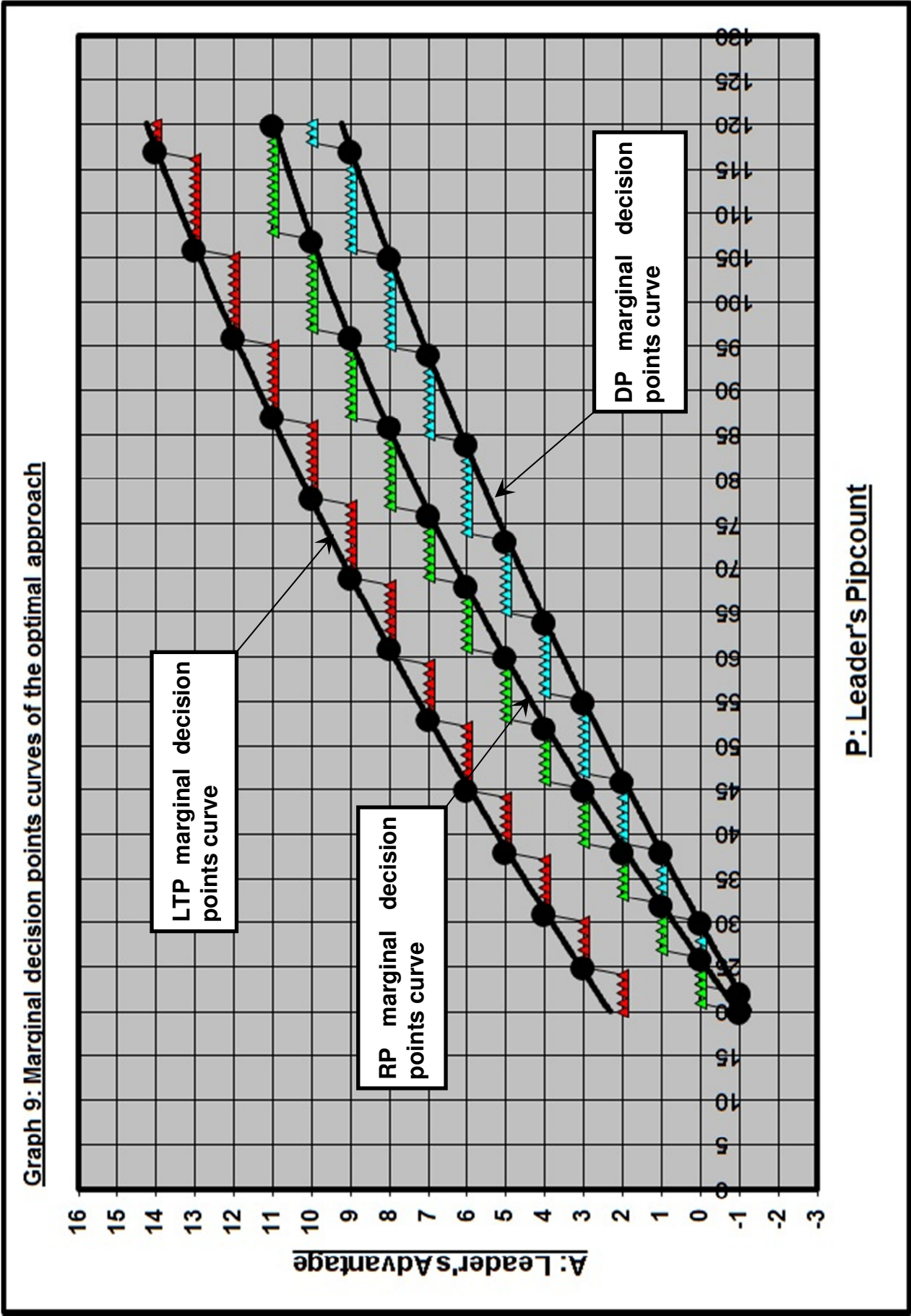


P: Leader's Pipcount

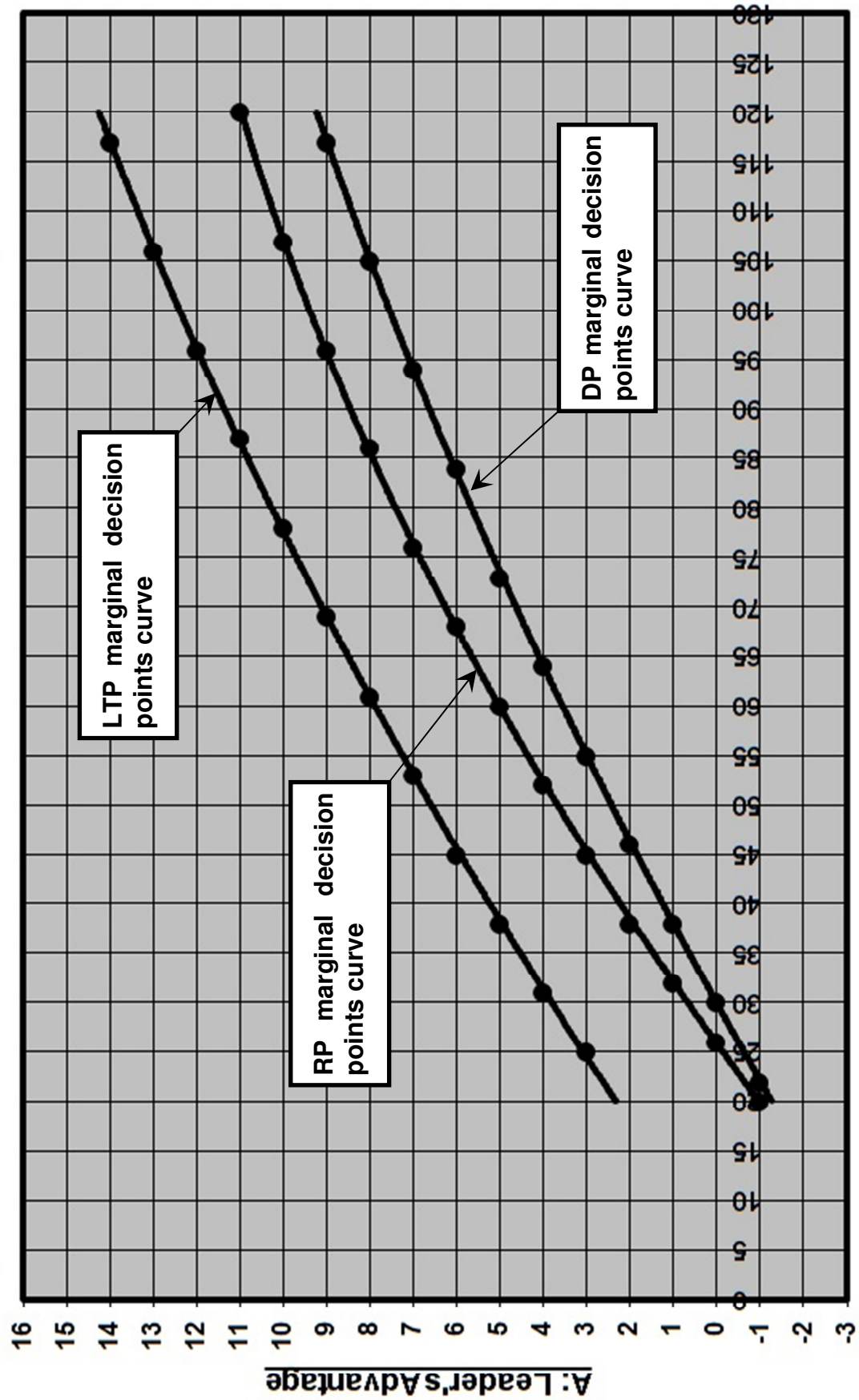
Graph 8: LTP curve of the optimal approach



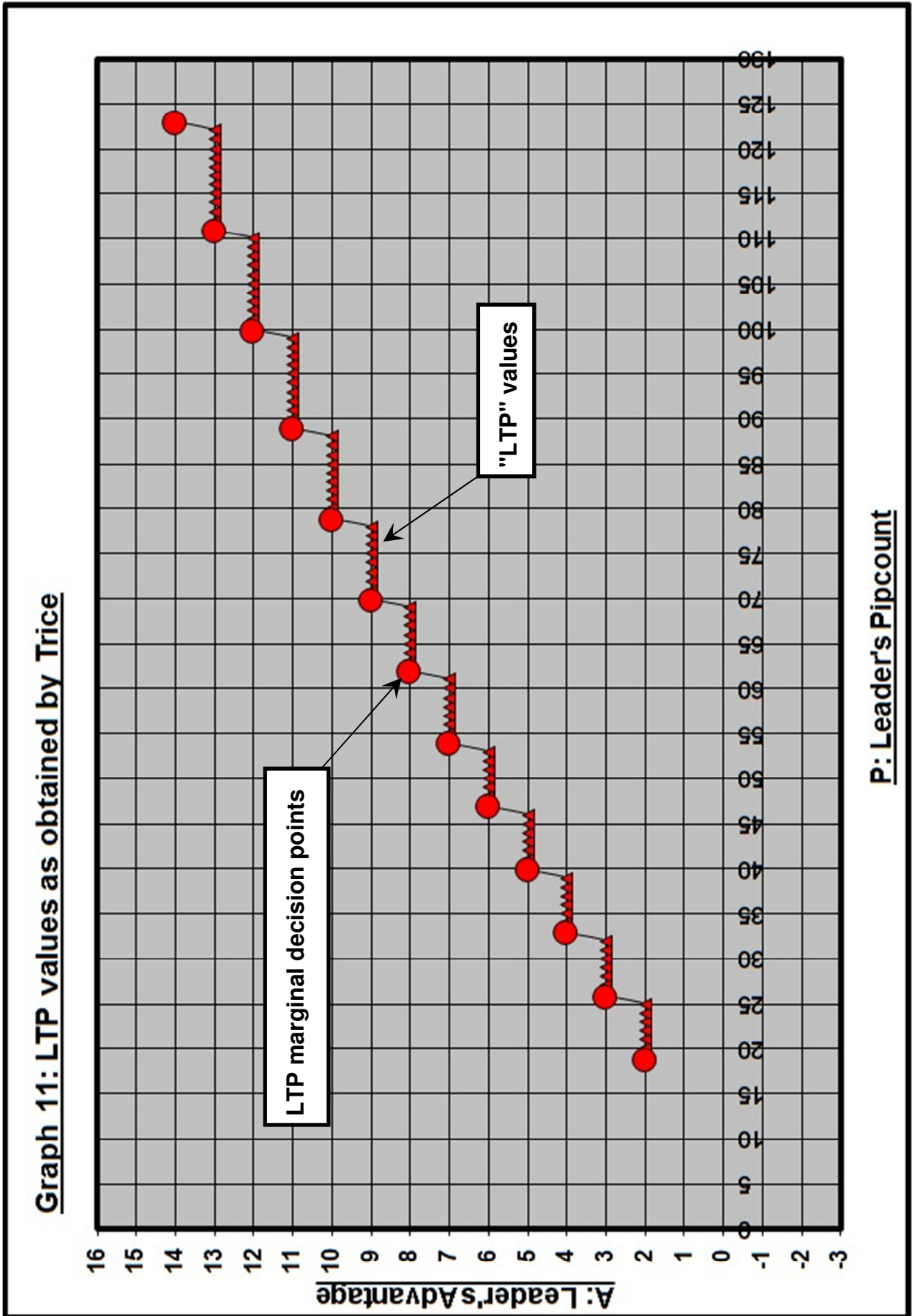
P: Leader's Pipcount



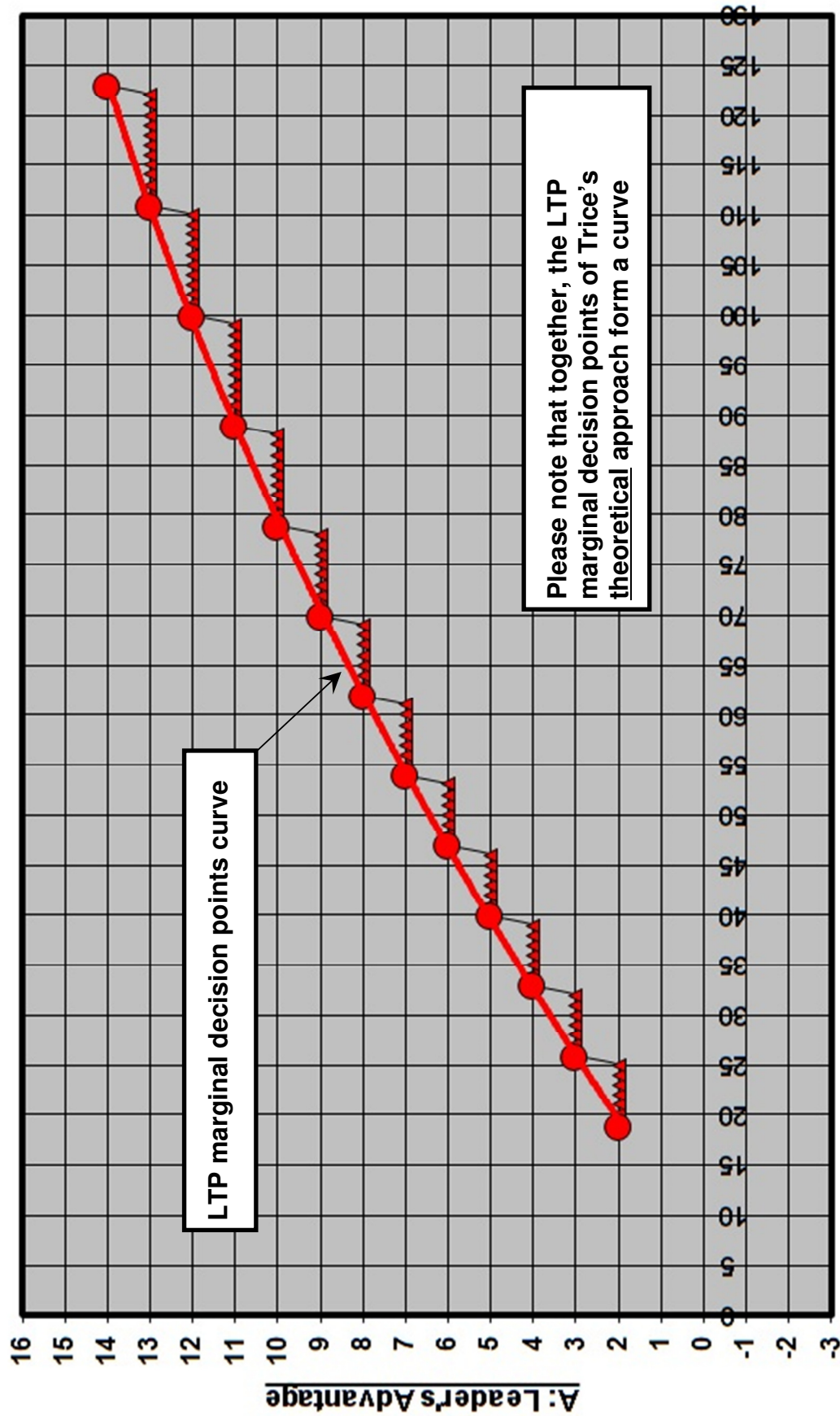
Graph 10: DP, RP and LTP marginal decision points curves of the optimal approach



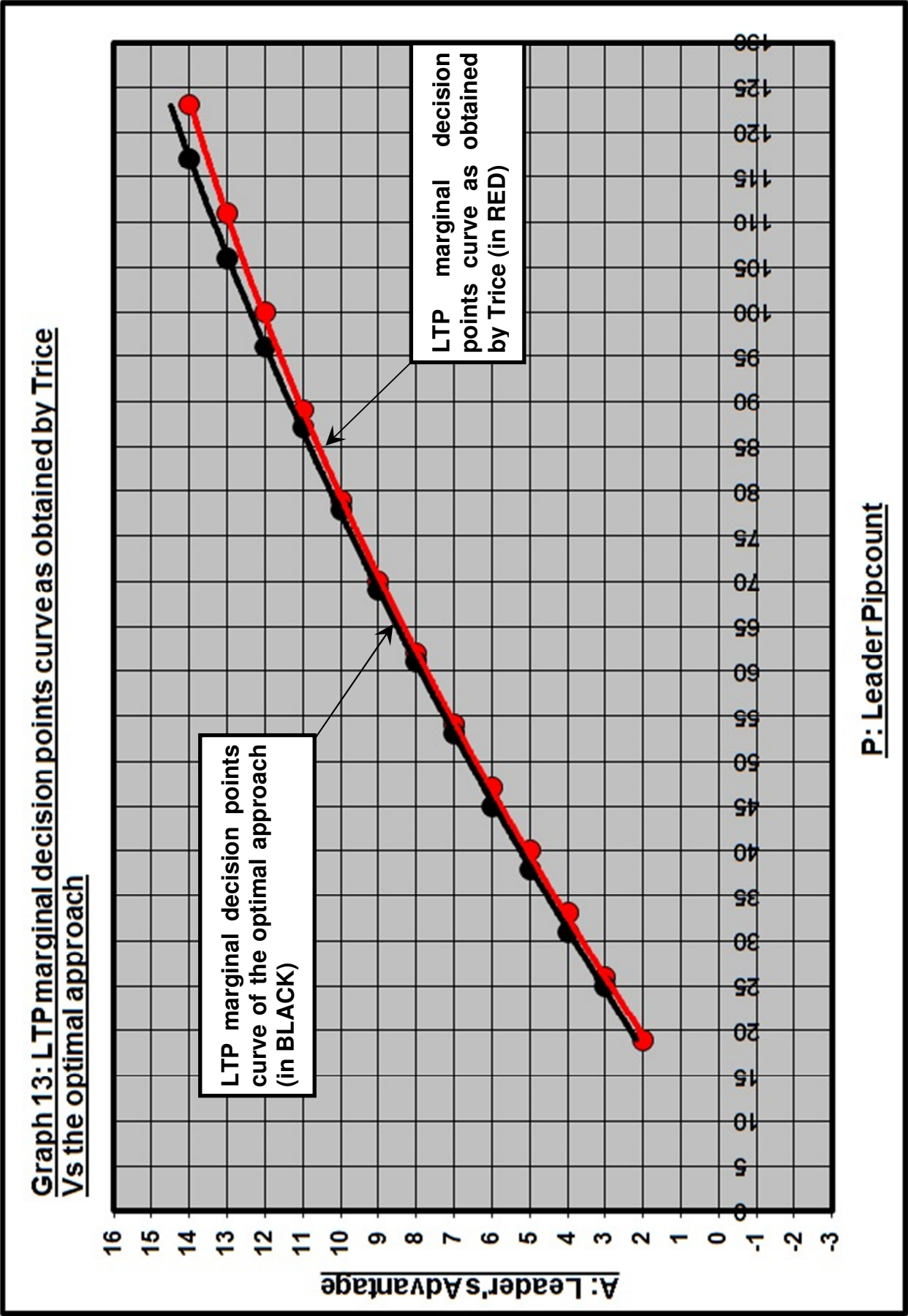
P: Leader's Pipcount



Graph 12: LTP marginal decision points curve as obtained by Trice



P: Leader's Pipcount

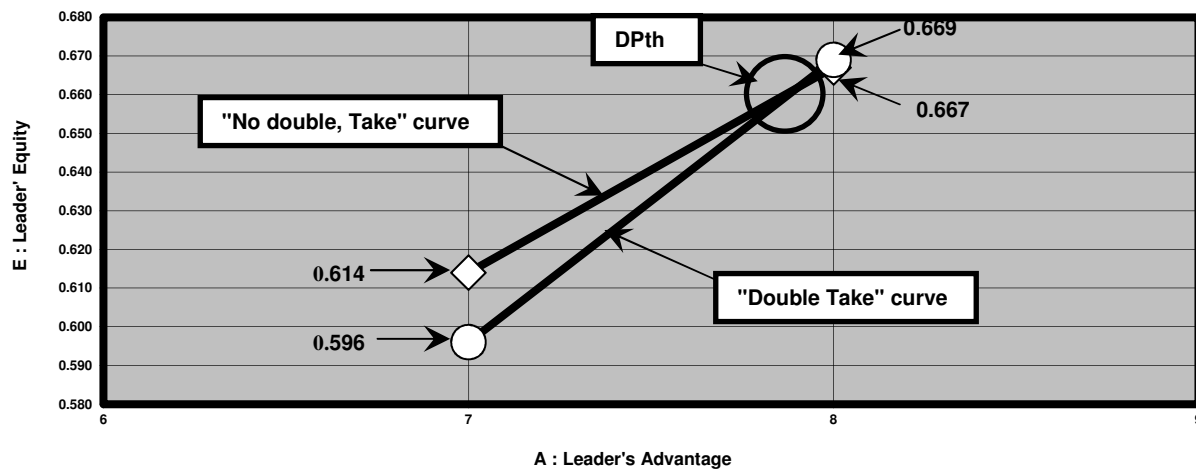


Appendix 1: Calculation of the DPth

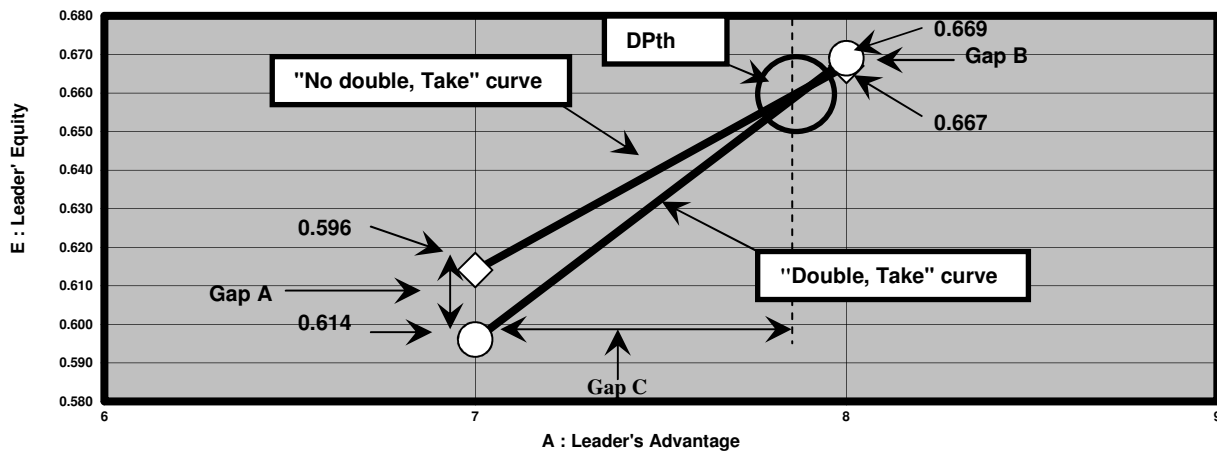
The data to be used are:

Leader's advantage	Equity No double, Take	Equity Double, Take	Proper Cube Handling
7	0.614	0.596	No double, Take
8	0.667	0.669	Double, Take

In the form of a graph, this gives:



The interpolation calculations are:



$$\text{Gap A} = 0.614 - 0.596 = 0.018$$

$$\text{Gap B} = 0.669 - 0.667 = 0.002$$

$$\text{Gap C} = (\text{Gap A}) / (\text{Gap A} + \text{Gap B})$$

$$\text{Gap C} = 0.018 / (0.018 + 0.002) = 0.0187 / 0.020 = 0.900$$

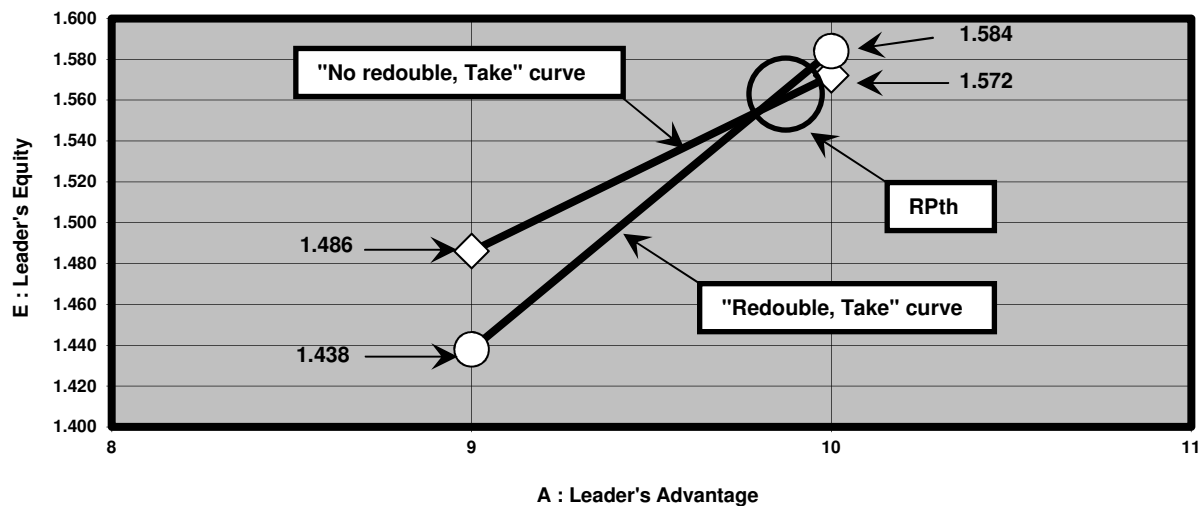
$$\text{DPth} = 7 \text{ pips} + \text{Gap C} = \underline{7.90 \text{ pips}}$$

Appendix 2: Calculation of the RPth

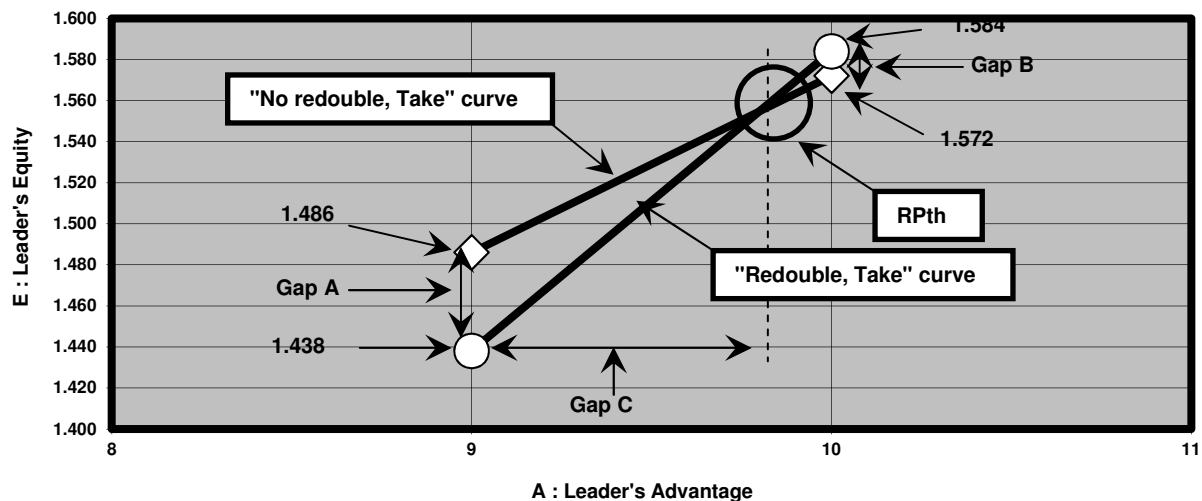
The data to be used are:

Leader's advantage	Equity No redouble, Take	Equity Redouble, Take	Proper Cube Handling
9	1.486	1.438	No redouble, Take
10	1.572	1.584	Redouble, Take

In the form of a graph, this gives:



The interpolation calculations are:



$$\text{Gap A} = 1.486 - 1.438 = 0.048$$

$$\text{Gap B} = 1.584 - 1.572 = 0.012$$

$$\text{Gap C} = (\text{Gap A}) / (\text{Gap A} + \text{Gap B})$$

$$\text{Gap C} = 0.048 / (0.048 + 0.012) = 0.048 / 0.060 = 0.800$$

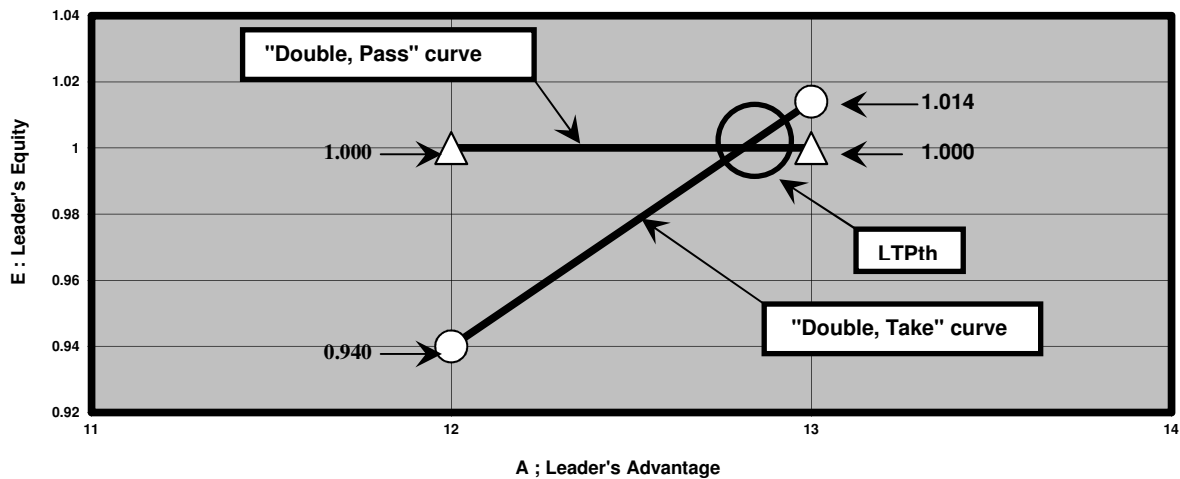
$$\text{RPth} = 9 \text{ pips} + \text{Gap C} = \underline{9.80 \text{ pips}}$$

Appendix 3: Calculation of the LTPth

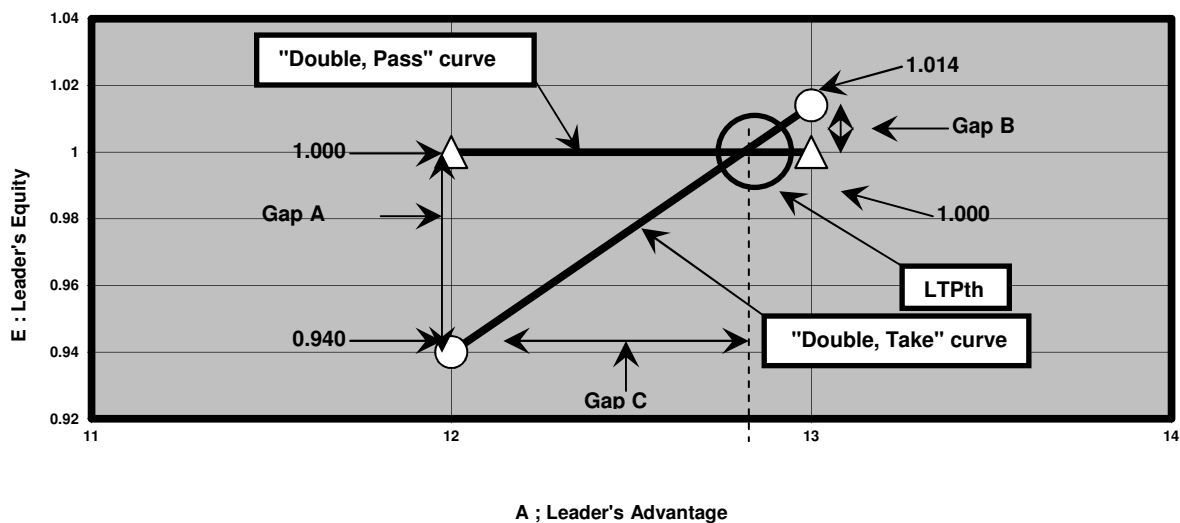
The data to be used comes from table 1 and it is presented as follows:

Leader's advantage	Equity Double, Take	Equity Double, Pass	Proper Cube Handling
12	0.940	1.000	Double, Take
13	1.014	1.000	Double, Pass

In the form of a graph, this gives:



The interpolation calculations are:



$$\text{Gap A} = 1.000 - 0.940 = 0.060$$

$$\text{Gap B} = 1.014 - 1.000 = 0.014$$

$$\text{Gap C} = (\text{Gap A}) / (\text{Gap A} + \text{Gap B})$$

$$\text{Gap C} = 0.060 / (0.060 + 0.014) = 0.060 / 0.074 = 0.810$$

$$\text{LTPth} = 12 \text{ pips} + \text{Gap C} = \underline{12.81 \text{ pips}}$$

Part 2: Analysis of some approaches

2.1 Methodology used

Previously, we saw that an approach can be presented using either one of the following techniques:

- Using criteria
- Using table presenting the DP, the RP and the LTP values
- Using graphs illustrating the DP, the RP and the LTP values
- Using a table presenting the MDP (Marginal Decision Points)
- Using a graph presenting the Marginal Decision Points curves

We also saw that whatever the technique used to describe an approach, the results obtained will always be exactly the same.

All approaches presented so far could be classified as follows:

- Old approaches
- Relatively recent approaches
- New approaches

The 8%, 9%, 12% approach and Thorp's approach are old ones.

Trice's approach is a relatively recent one.

The new approaches are the following:

- Optimal
- Chabot

In the second part of this article, we will analyze the following approaches:

- Optimal
- 8%,9%,12%
- Thorp
- Trice
- Chabot

Each approach will be analyzed by using the exact same methodology. The methodology used includes the following steps:

- 1) The first step is a short history presentation about the elaborated approach.
- 2) The second step consists of presenting the criteria to use.
- 3) The third step consists of presenting all the values for a leader's pipcount ranging from 20 pips to 120 pips in the form of a table.
- 4) The fourth step consists of presenting all the values for a leader's pipcount ranging from 20 pips to 120 pips in a form of a graph.

- 5) The fifth step consists of presenting the marginal decision points in a table form.
- 6) The sixth step consists of presenting the obtained results when "P" (the leader's pipcount) is 100 pips and when "P" is 50 pips.
- 7) The seventh step consists of calculating the precision of an approach. To obtain the desired result expressed in percentage (%), the following technique is used:
 - 7.1) The DP, the RP and the LTP for P ranging from 20 pips to 120 pips are presented. From 20 pips to 120 pips, there are 101 pips. Given that for every pip, there are 3 practical results; this implies that there is a grand total of 303 results.
 - 7.2) All results of the approach analyzed are compared, one by one, with the obtained results with the optimal approach. To obtain a good result, the results of the analyzed approach must necessarily be identical to the results of the optimal approach.
 - 7.3) The total number of good results is obtained by the adding all the good results obtained.
 - 7.4) The precision of an approach is expressed in percentage (%) of good results obtained, with regards to the 303 obtained results of the optimal approach.
- 8) The eighth step consists of comparing the marginal decision points curves of the approach analyzed with the marginal decision points curves of the optimal approach.
- 9) The ninth step consists of recommending whether to use the analysed approach or not based on the obtained precision with regards to all the analyzed approaches.
- 10) And finally, the last step consists of comparing how easy it is to remember the analyzed approach with regards to all the analyzed approaches.

2.2 Optimal approach

The optimal approach is a new theoretical approach. This approach was elaborated in great detail in part 1 of this article.

Table 2.1 presents the obtained values.

Graph 2.1 illustrates the obtained values.

Table 2.2 presents the marginal decision points.

Graph 2.2 illustrates the marginal decision points curves.

Using this approach, when "P" (the leader's pipcount) is 100 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 108 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 110 pips.
- The trailer must take if his pipcount is equal or inferior to 112 pips.

Using this approach, when "P" (the leader's pipcount) is 50 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 53 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 54 pips.
- The trailer must take if his pipcount is equal or inferior to 56 pips.

Since the optimal approach must be considered as the "reference approach", this implies that the precision of this approach is obviously 100%.

With regards to all approaches analyzed, this approach produces perfect results. Consequently, this approach is obviously a recommendable one.

With regards to all the analyzed approaches, this approach is difficult to remember. To use this approach you must memorize the table of marginal decision points as presented in table 2.2.

* * * * *

List of the tables and graphs of section 2.2

Table 2.1: Obtained values for the optimal approach

Table 2.2: Marginal Decision Points of the optimal approach

Graph 2.1: Obtained values for the optimal approach

Graph 2.2: Marginal decision points curves of the optimal approach

Table 2.1: Obtained values for the optimal approach

P	DP	RP	LTP
20	-1	-1	2
21	-1	0	2
22	-1	0	2
23	0	0	2
24	0	0	2
25	0	0	3
26	0	0	3
27	0	1	3
28	0	1	3
29	0	1	3
30	0	1	3
31	1	1	4
32	1	1	4
33	1	2	4
34	1	2	4
35	1	2	4
36	1	2	4
37	1	2	4
38	1	2	5
39	2	3	5
40	2	3	5
41	2	3	5
42	2	3	5
43	2	3	5
44	2	3	5
45	2	3	6
46	2	4	6
47	3	4	6
48	3	4	6
49	3	4	6
50	3	4	6
51	3	4	6
52	3	4	6
53	3	5	7
54	3	5	7
55	3	5	7
56	4	5	7
57	4	5	7
58	4	5	7
59	4	5	7
60	4	5	7
61	4	6	8
62	4	6	8
63	4	6	8
64	4	6	8
65	5	6	8
66	5	6	8
67	5	6	8
68	5	6	8
69	5	7	9
70	5	7	9

P	DP	RP	LTP
71	5	7	9
72	5	7	9
73	5	7	9
74	6	7	9
75	6	7	9
76	6	7	9
77	6	8	9
78	6	8	10
79	6	8	10
80	6	8	10
81	6	8	10
82	6	8	10
83	6	8	10
84	6	8	10
85	7	8	10
86	7	8	10
87	7	9	11
88	7	9	11
89	7	9	11
90	7	9	11
91	7	9	11
92	7	9	11
93	7	9	11
94	7	9	11
95	8	9	11
96	8	9	12
97	8	10	12
98	8	10	12
99	8	10	12
100	8	10	12
101	8	10	12
102	8	10	12
103	8	10	12
104	8	10	12
105	8	10	12
106	9	10	13
107	9	10	13
108	9	11	13
109	9	11	13
110	9	11	13
111	9	11	13
112	9	11	13
113	9	11	13
114	9	11	13
115	9	11	13
116	9	11	13
117	9	11	14
118	10	11	14
119	10	11	14
120	10	11	14

Table 2.2: Marginal Decision Points of the optimal approach

MDP	DP
22	-1
30	0
38	1
46	2
55	3
64	4
73	5
84	6
94	7
105	8
117	9

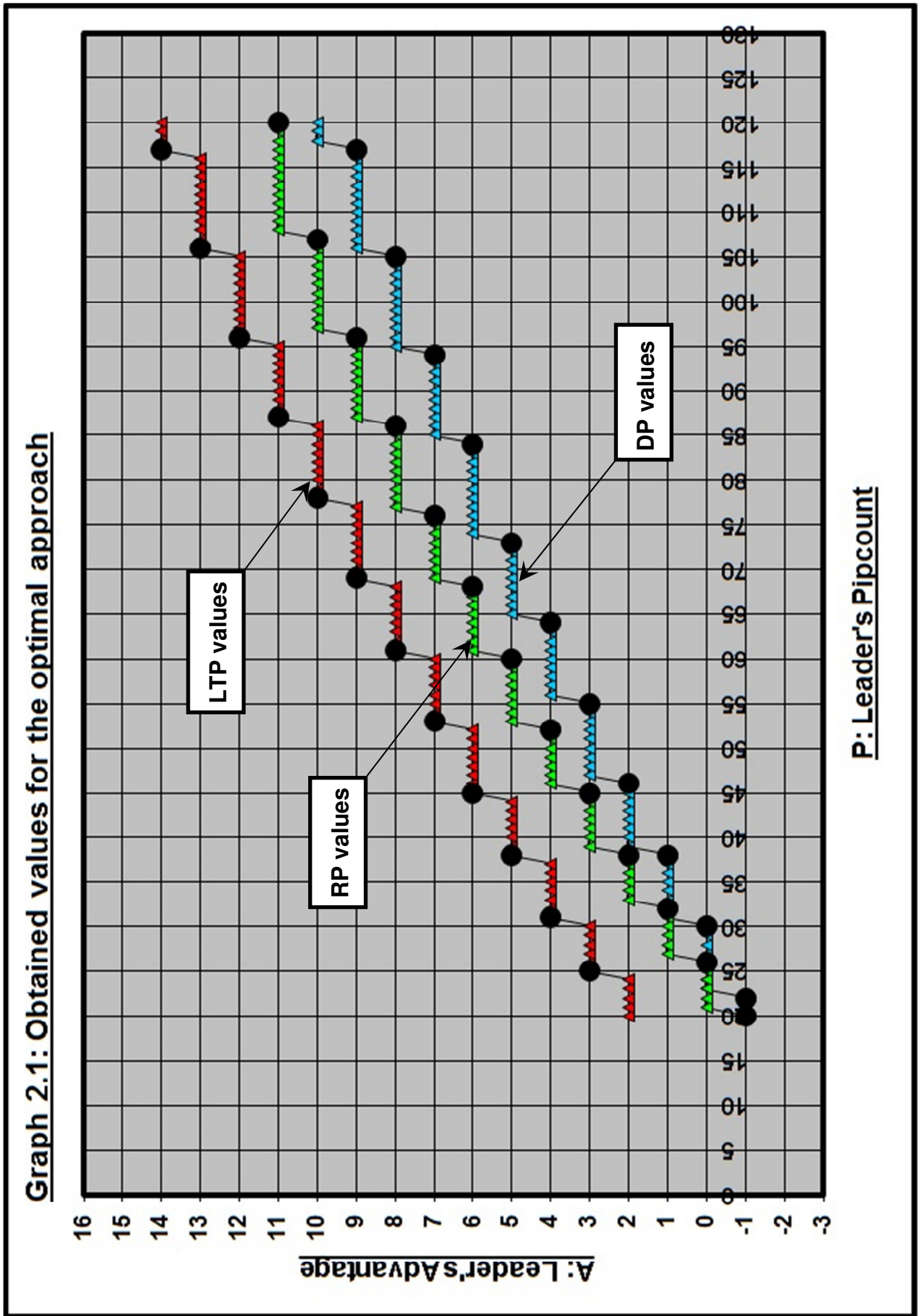
MDP	RP
20	-1
26	0
32	1
38	2
45	3
52	4
60	5
68	6
76	7
86	8
96	9
107	10
120	11

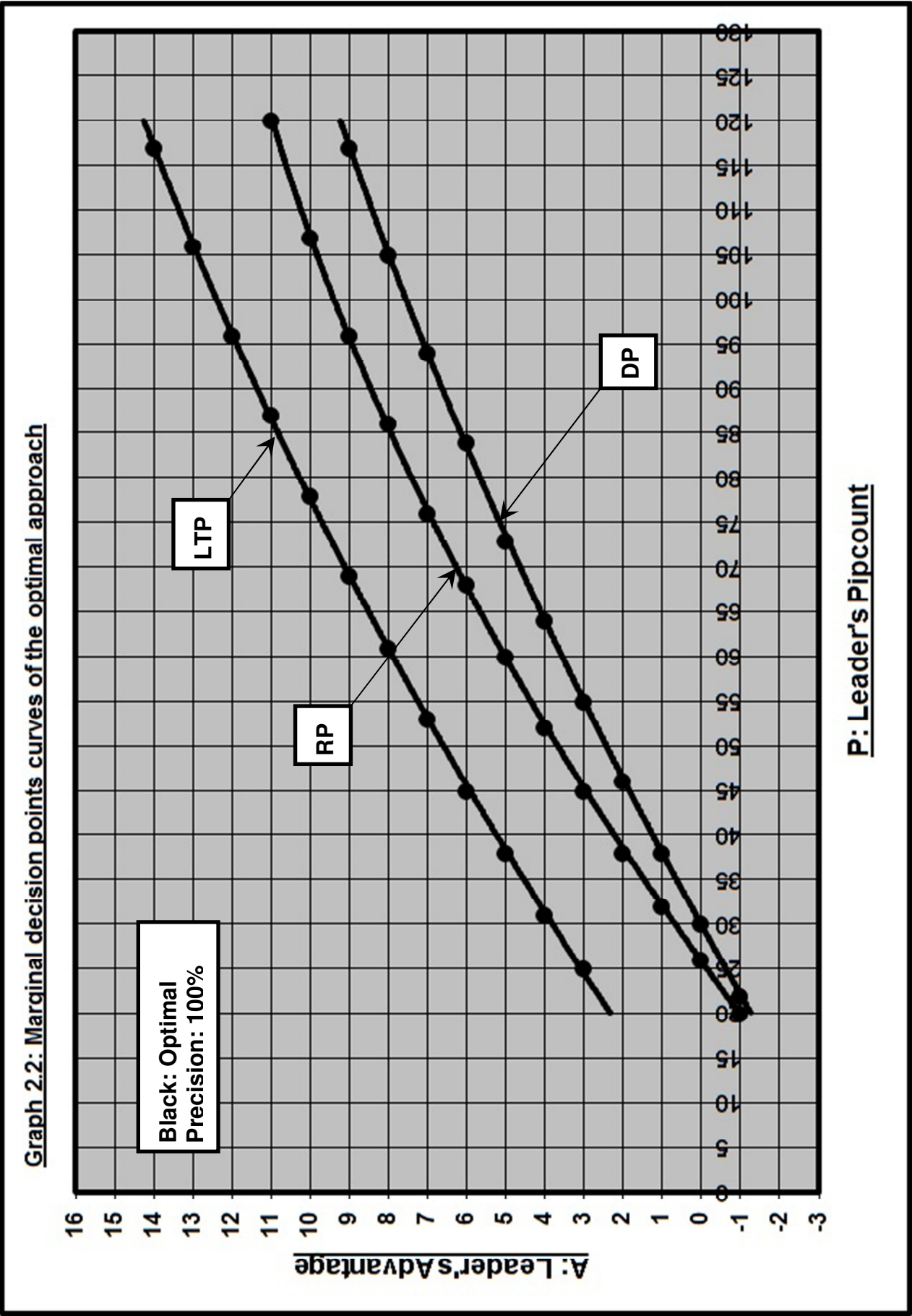
MDP	LTP
25	3
31	4
38	5
45	6
53	7
61	8
69	9
78	10
87	11
96	12
106	13
117	14

To decide if the leader must double, use MDP which is equal or immediately above the leader's pipcount; the corresponding DP result is the minimum advantage required to double.

To decide if the leader must redouble, use MDP which is equal or immediately above the leader's pipcount; the corresponding RP result is the minimum advantage required to redouble.

To decide if the trailer must take, use MDP which is equal or immediately below the leader's pipcount; the corresponding LTP result is the maximum disadvantage required to take.





2.3 8 %, 9%, 12% approach

The 8%, 9%, 12% approach is an "old" approach which appeared in the 1970s. The author of this approach is unknown.

The criteria of this approach are:

DP = (P x 8%), up

RP = (P x 9%), up

LTP = (P x 12%), down

Table 3.1 presents the obtained values.

Graph 3.1 illustrates the obtained values.

Table 3.2 presents the marginal decision points.

Using this approach, when "P" (the leader's pipcount) is 100 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 108 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 109 pips.
- The trailer must take if his pipcount is equal or inferior to 112 pips.

Using this approach, when "P" (the leader's pipcount) is 50 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 54 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 55 pips.
- The trailer must take if his pipcount is equal or inferior to 56 pips.

Table 3.3 presents the calculations of the precision. The results obtained are:

Good results: 40.9%

Results with a 1-pip difference: 41.6%

Results with a 2-pip difference: 12.9%

Results with a 3-pip difference: 4.6%

Results with a 4-pip or more difference: 0.0%

Graph 3.2 compares the marginal decision points curves of this approach to the optimal approach.

With regards to all the analyzed approaches, this approach does not produce precise enough results. Consequently, this approach isn't recommendable and evaluating whether or not it is easy to remember is not relevant.

* * * * *

List of the tables and graphs of section 2.3

Table 3.1:	Obtained values for the 8%, 9%, 12% approach
Table 3.2:	Marginal decision points of the 8%, 9%, 12% approach
Table 3.3:	Calculation of the precision of the 8%, 9%, 12% approach
Graph 3.1:	Obtained values for the 8%, 9%, 12% approach
Graph 3.2:	Marginal decision points curves of the 8%, 9%, 12% approach Vs the optimal approach

Table 3.1: Obtained values for the 8%, 9%, 12% approach

P	DP	RP	LTP
20	2	2	2
21	2	2	2
22	2	2	2
23	2	3	2
24	2	3	2
25	2	3	3
26	3	3	3
27	3	3	3
28	3	3	3
29	3	3	3
30	3	3	3
31	3	3	3
32	3	3	3
33	3	3	3
34	3	4	4
35	3	4	4
36	3	4	4
37	3	4	4
38	4	4	4
39	4	4	4
40	4	4	4
41	4	4	4
42	4	4	5
43	4	4	5
44	4	4	5
45	4	5	5
46	4	5	5
47	4	5	5
48	4	5	5
49	4	5	5
50	4	5	6
51	5	5	6
52	5	5	6
53	5	5	6
54	5	5	6
55	5	5	6
56	5	6	6
57	5	6	6
58	5	6	6
59	5	6	7
60	5	6	7
61	5	6	7
62	5	6	7
63	6	6	7
64	6	6	7
65	6	6	7
66	6	6	7
67	6	7	8
68	6	7	8
69	6	7	8
70	6	7	8

P	DP	RP	LTP
71	6	7	8
72	6	7	8
73	6	7	8
74	6	7	8
75	6	7	9
76	7	7	9
77	7	7	9
78	7	8	9
79	7	8	9
80	7	8	9
81	7	8	9
82	7	8	9
83	7	8	9
84	7	8	10
85	7	8	10
86	7	8	10
87	7	8	10
88	8	8	10
89	8	9	10
90	8	9	10
91	8	9	10
92	8	9	11
93	8	9	11
94	8	9	11
95	8	9	11
96	8	9	11
97	8	9	11
98	8	9	11
99	8	9	11
100	8	9	12
101	9	10	12
102	9	10	12
103	9	10	12
104	9	10	12
105	9	10	12
106	9	10	12
107	9	10	12
108	9	10	12
109	9	10	13
110	9	10	13
111	9	10	13
112	9	11	13
113	10	11	13
114	10	11	13
115	10	11	13
116	10	11	13
117	10	11	14
118	10	11	14
119	10	11	14
120	10	11	14

Table 3.2: Marginal Decision Points of the 8%, 9%, 12% approach

MDP	DP
25	2
37	3
50	4
62	5
75	6
87	7
100	8
112	9

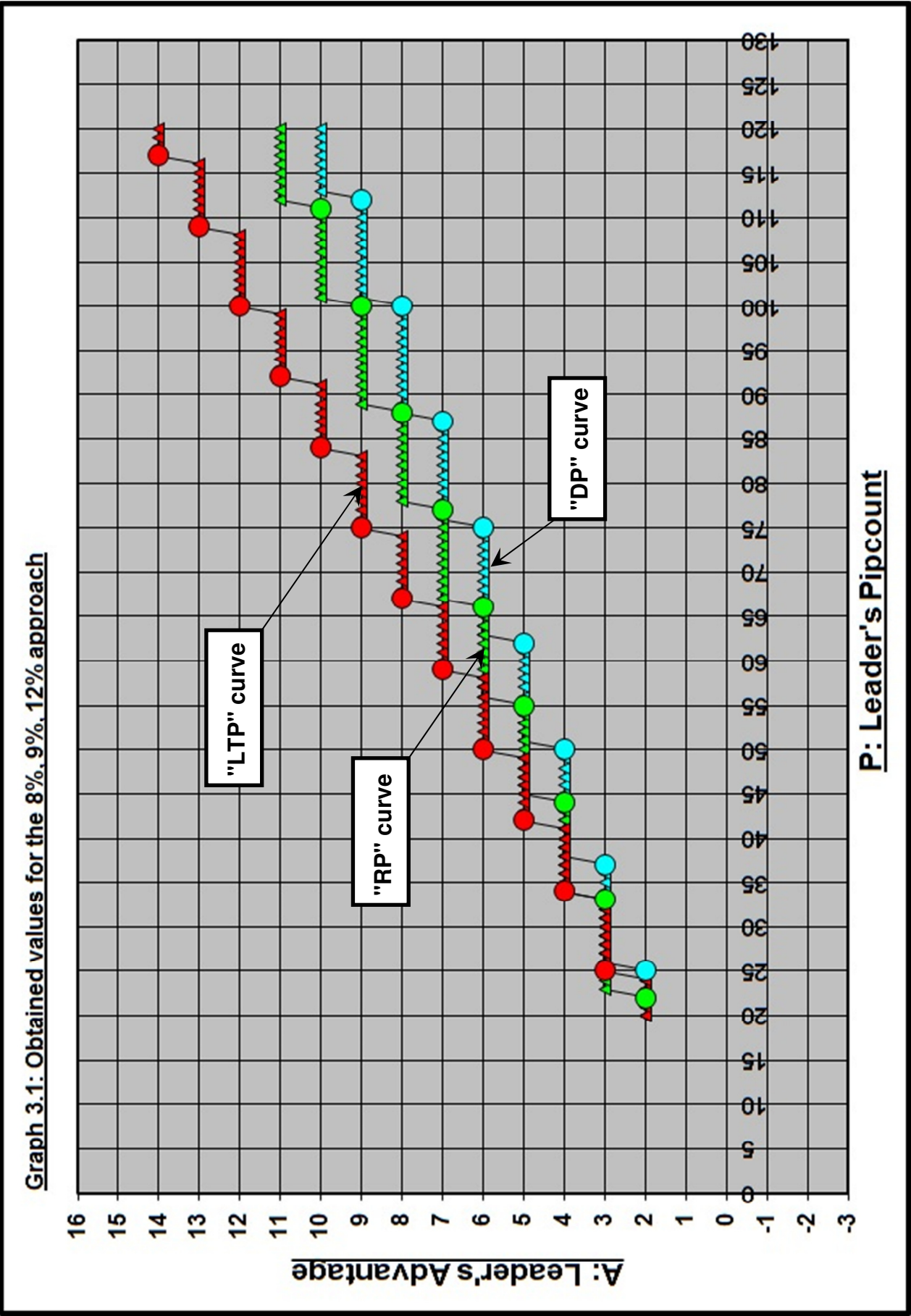
MDP	RP
22	2
33	3
44	4
55	5
66	6
77	7
88	8
100	9
111	10

MDP	LTP
25	3
34	4
42	5
50	6
59	7
67	8
75	9
84	10
92	11
100	12
109	13
117	14

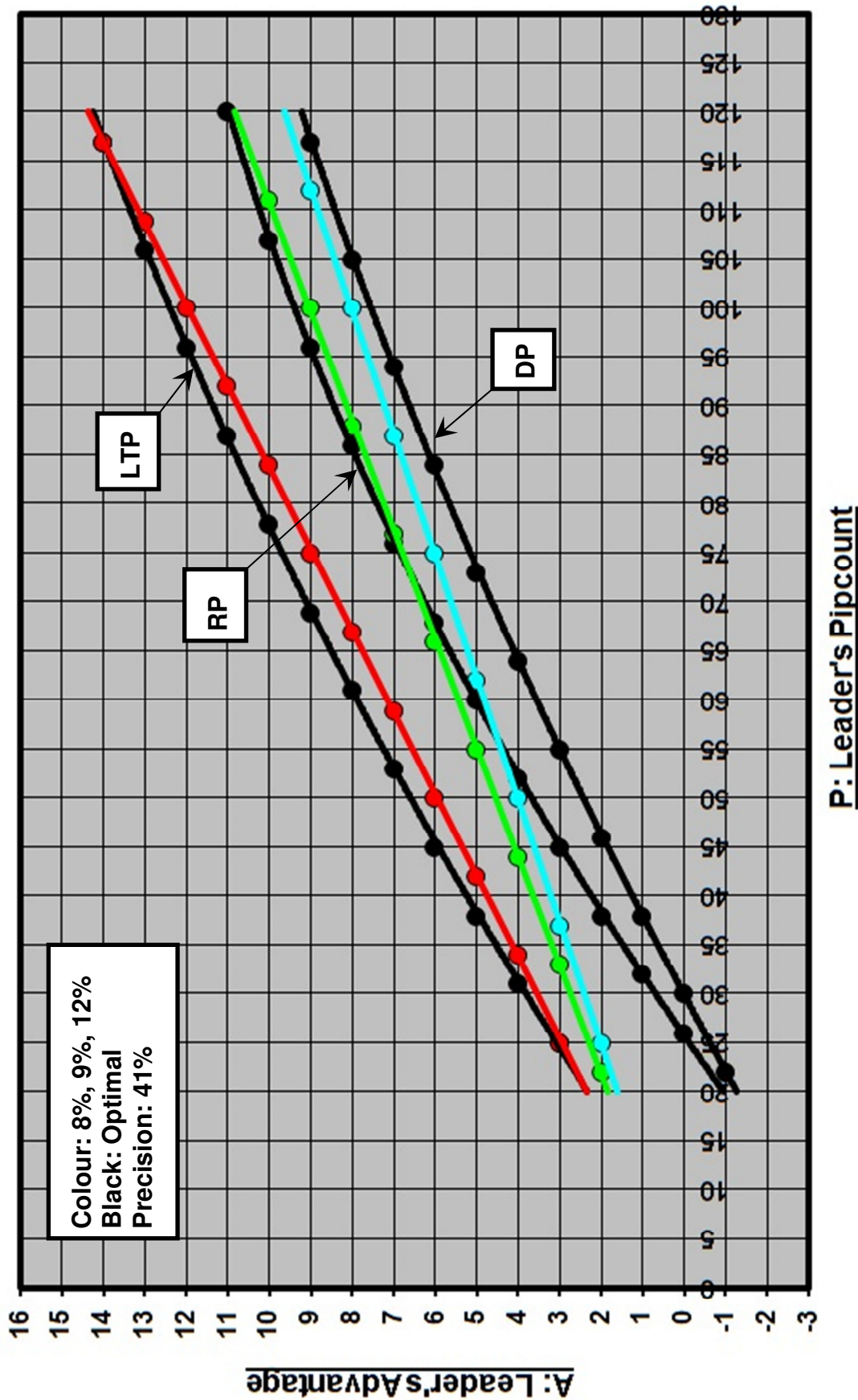
Table 3.3: Calculation of the precision of the 8%, 9%, 12% approach

P	DP opt	DP obtained	Gap	RP opt	RP obtained	GAP	LTP opt	LTP obtained	Gap
20	-1	2	3	-1	2	3	2	2	0
21	-1	2	3	0	2	2	2	2	0
22	-1	2	3	0	2	2	2	2	0
23	0	2	2	0	3	3	2	2	0
24	0	2	2	0	3	3	2	2	0
25	0	2	2	0	3	3	3	3	0
26	0	3	3	0	3	3	3	3	0
27	0	3	3	1	3	2	3	3	0
28	0	3	3	1	3	2	3	3	0
29	0	3	3	1	3	2	3	3	0
30	0	3	3	1	3	2	3	3	0
31	1	3	2	1	3	2	4	3	1
32	1	3	2	1	3	2	4	3	1
33	1	3	2	2	3	1	4	3	1
34	1	3	2	2	4	2	4	4	0
35	1	3	2	2	4	2	4	4	0
36	1	3	2	2	4	2	4	4	0
37	1	3	2	2	4	2	4	4	0
38	1	4	3	2	4	2	5	4	1
39	2	4	2	3	4	1	5	4	1
40	2	4	2	3	4	1	5	4	1
41	2	4	2	3	4	1	5	4	1
42	2	4	2	3	4	1	5	5	0
43	2	4	2	3	4	1	5	5	0
44	2	4	2	3	4	1	5	5	0
45	2	4	2	3	5	2	6	5	1
46	2	4	2	4	5	1	6	5	1
47	3	4	1	4	5	1	6	5	1
48	3	4	1	4	5	1	6	5	1
49	3	4	1	4	5	1	6	5	1
50	3	4	1	4	5	1	6	5	0
51	3	5	2	4	5	1	6	5	0
52	3	5	2	4	5	1	6	6	0
53	3	5	2	5	5	0	7	6	1
54	3	5	2	5	5	0	7	6	1
55	3	5	2	5	5	0	7	6	1
56	4	5	1	5	6	1	7	6	1
57	4	5	1	5	6	1	7	6	1
58	4	5	1	5	6	1	7	6	1
59	4	5	1	5	6	1	7	7	0
60	4	5	1	5	6	1	7	7	0
61	4	5	1	6	6	0	8	7	1
62	4	5	1	6	6	0	8	7	1
63	4	6	2	6	6	0	8	7	1
64	4	6	2	6	6	0	8	7	1
65	5	6	1	6	6	0	8	7	1
66	5	6	1	6	6	0	8	7	1
67	5	6	1	6	7	1	8	8	0
68	5	6	1	6	7	1	8	8	0
69	5	6	1	7	7	0	9	8	1
70	5	6	1	7	7	0	9	8	1
71	5	6	1	7	7	0	9	8	1
72	5	6	1	7	7	0	9	8	1
73	5	6	1	7	7	0	9	8	1
74	6	6	0	7	7	0	9	8	1
75	6	6	0	7	7	0	9	9	0
76	6	7	1	7	7	0	9	9	0
77	6	7	1	8	7	1	9	9	0
78	6	7	1	8	8	0	10	9	1
79	6	7	1	8	8	0	10	9	1
80	6	7	1	8	8	0	10	9	1
81	6	7	1	8	8	0	10	9	1
82	6	7	1	8	8	0	10	9	1
83	6	7	1	8	8	0	10	9	1
84	6	7	1	8	8	0	10	10	0
85	7	7	0	8	8	0	10	10	0
86	7	7	0	8	8	0	10	10	0
87	7	7	0	9	8	1	11	10	1
88	7	8	1	9	8	1	11	10	1
89	7	8	1	9	9	0	11	10	1
90	7	8	1	9	9	0	11	10	1
91	7	8	1	9	9	0	11	10	1
92	7	8	1	9	9	0	11	11	0
93	7	8	1	9	9	0	11	11	0
94	7	8	1	9	9	0	11	11	0
95	8	8	0	9	9	0	11	11	0
96	8	8	0	9	9	0	12	11	1
97	8	8	0	10	9	1	12	11	1
98	8	8	0	10	9	1	12	11	1
99	8	8	0	10	9	1	12	11	1
100	8	8	0	10	9	1	12	12	0
101	8	9	1	10	10	0	12	12	0
102	8	9	1	10	10	0	12	12	0
103	8	9	1	10	10	0	12	12	0
104	8	9	1	10	10	0	12	12	0
105	8	9	1	10	10	0	12	12	0
106	9	9	0	10	10	0	13	12	1
107	9	9	0	10	10	0	13	12	1
108	9	9	0	11	10	1	13	12	1
109	9	9	0	11	10	1	13	13	0
110	9	9	0	11	10	1	13	13	0
111	9	9	0	11	10	1	13	13	0
112	9	9	0	11	11	0	13	13	0
113	9	10	1	11	11	0	13	13	0
114	9	10	1	11	11	0	13	13	0
115	9	10	1	11	11	0	13	13	0
116	9	10	1	11	11	0	13	13	0
117	9	10	1	11	11	0	14	14	0
118	10	10	0	11	11	0	14	14	0
119	10	10	0	11	11	0	14	14	0
120	10	10	0	11	11	0	14	14	0

Good results	124 on 303 = 40.9%
Results with a 1-pip difference	126 on 303 = 41.6%
Results with a 2-pip difference	39 on 303 = 12.9%
Results with a 3-pip difference	14 on 303 = 4.6%



**Graph 3.2: Marginal decision points curves of the 8%, 9%, 12% approach
Vs the optimal approach**



2.4 Thorp's approach

Thorp's approach is an "old" approach which was presented in the 1970s. The author of this approach is Edward O. Thorp. Mr. Thorp, who holds a Ph.D. in mathematics, also wrote a very well-known book on blackjack game, entitled "Beat the Dealer: A Winning Strategy for the Game of Twenty-One", which was a best-seller.

The criteria of this approach are:

$$\begin{aligned} DP &= ((P \times 10\%) - 2), \text{ up} \\ RP &= ((P \times 10\%) - 1), \text{ up} \\ LTP &= ((P \times 10\%) + 2), \text{ down} \end{aligned}$$

Table 4.1 presents the obtained values.

Graph 4.1 illustrates the obtained values.

Table 4.2 presents the marginal decision points.

Using this approach, when "P" (the leader's pipcount) is 100 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 108 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 109 pips.
- The trailer must take if his pipcount is equal or inferior to 112 pips.

Using this approach, when "P" (the leader's pipcount) is 50 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 53 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 54 pips.
- The trailer must take if his pipcount is equal or inferior to 57 pips.

Table 4.3 presents the calculations of the precision. The results obtained are:

- Good results: 52.5%
- Results with a 1-pip difference: 41.9%
- Results with a 2-pip difference: 5.6%
- Results with a 3-pip or more difference: 0.0%

Graph 4.2 compares the marginal decision points curves of this approach to the optimal approach.

With regards to all the analyzed approaches, this approach does not produce precise enough results. Consequently, this approach isn't recommendable and evaluating whether or not it is easy to remember is not relevant.

* * * * *

List of the tables and graphs of section 2.4

Table 4.1: **Obtained values for Thorp's approach**

Table 4.2: **Marginal Decision Points of Thorp's approach**

Table 4.3: **Calculation of the precision of Thorp's approach**

Graph 4.1: **Obtained values for Thorp's approach**

Graph 4.2: **Marginal decision points curves of Thorp's approach Vs the optimal approach**

Table 4.1: Obtained values for Thorp's approach

P	DP	RP	LTP
20	0	1	4
21	1	2	4
22	1	2	4
23	1	2	4
24	1	2	4
25	1	2	4
26	1	2	4
27	1	2	4
28	1	2	4
29	1	2	4
30	1	2	5
31	2	3	5
32	2	3	5
33	2	3	5
34	2	3	5
35	2	3	5
36	2	3	5
37	2	3	5
38	2	3	5
39	2	3	5
40	2	3	6
41	3	4	6
42	3	4	6
43	3	4	6
44	3	4	6
45	3	4	6
46	3	4	6
47	3	4	6
48	3	4	6
49	3	4	6
50	3	4	7
51	4	5	7
52	4	5	7
53	4	5	7
54	4	5	7
55	4	5	7
56	4	5	7
57	4	5	7
58	4	5	7
59	4	5	7
60	4	5	8
61	5	6	8
62	5	6	8
63	5	6	8
64	5	6	8
65	5	6	8
66	5	6	8
67	5	6	8
68	5	6	8
69	5	6	8
70	5	6	9

P	DP	RP	LTP
71	6	7	9
72	6	7	9
73	6	7	9
74	6	7	9
75	6	7	9
76	6	7	9
77	6	7	9
78	6	7	9
79	6	7	9
80	6	7	10
81	7	8	10
82	7	8	10
83	7	8	10
84	7	8	10
85	7	8	10
86	7	8	10
87	7	8	10
88	7	8	10
89	7	8	10
90	7	8	11
91	8	9	11
92	8	9	11
93	8	9	11
94	8	9	11
95	8	9	11
96	8	9	11
97	8	9	11
98	8	9	11
99	8	9	11
100	8	9	12
101	9	10	12
102	9	10	12
103	9	10	12
104	9	10	12
105	9	10	12
106	9	10	12
107	9	10	12
108	9	10	12
109	9	10	12
110	9	10	13
111	10	11	13
112	10	11	13
113	10	11	13
114	10	11	13
115	10	11	13
116	10	11	13
117	10	11	13
118	10	11	13
119	10	11	13
120	10	11	14

Table 4.2: Marginal Decision Points of Thorp's approach

MDP	DP
20	0
30	1
40	2
50	3
60	4
70	5
80	6
90	7
100	8
110	9
120	10

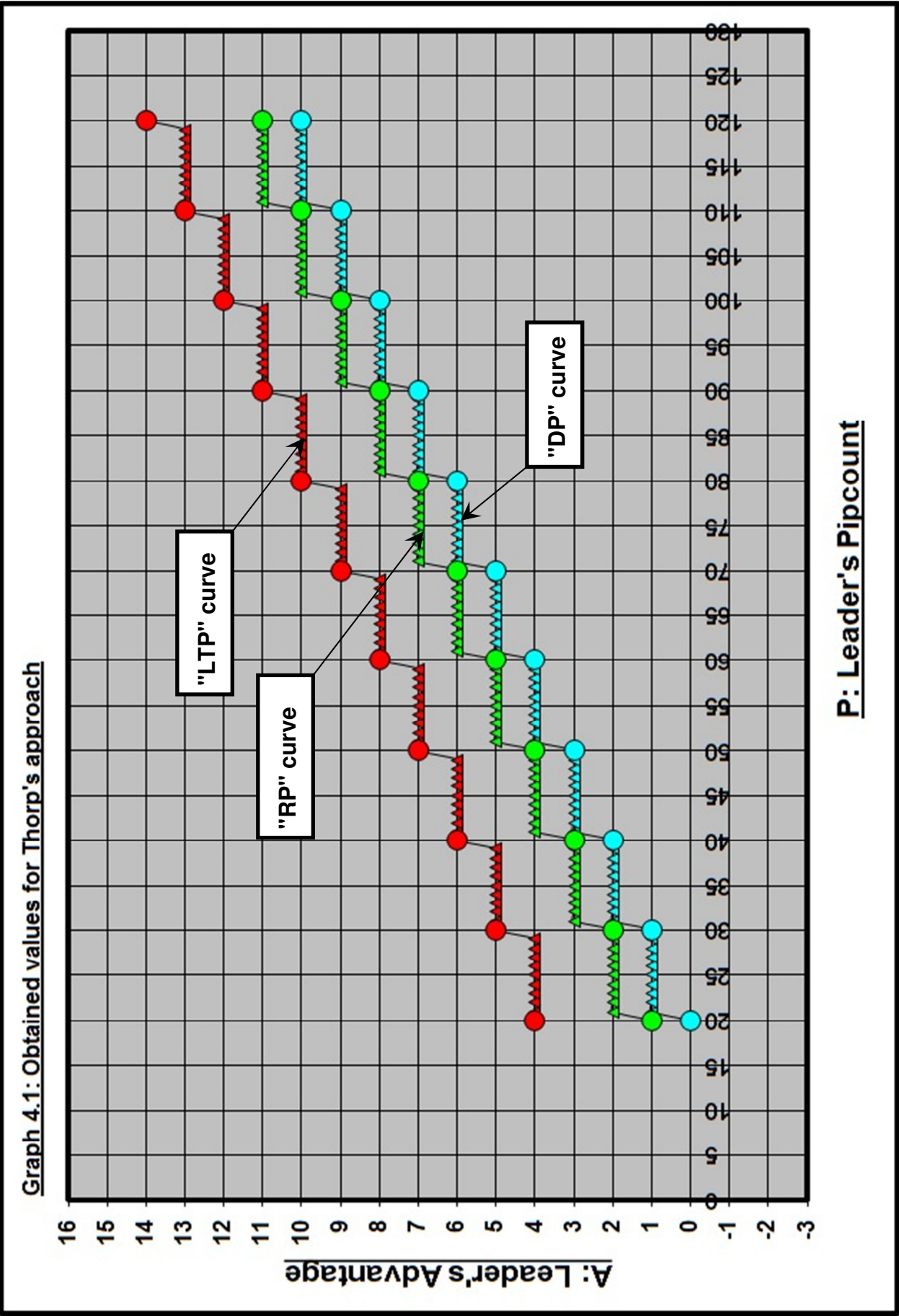
MDP	RP
20	1
30	2
40	3
50	4
60	5
70	6
80	7
90	8
100	9
110	10
120	11

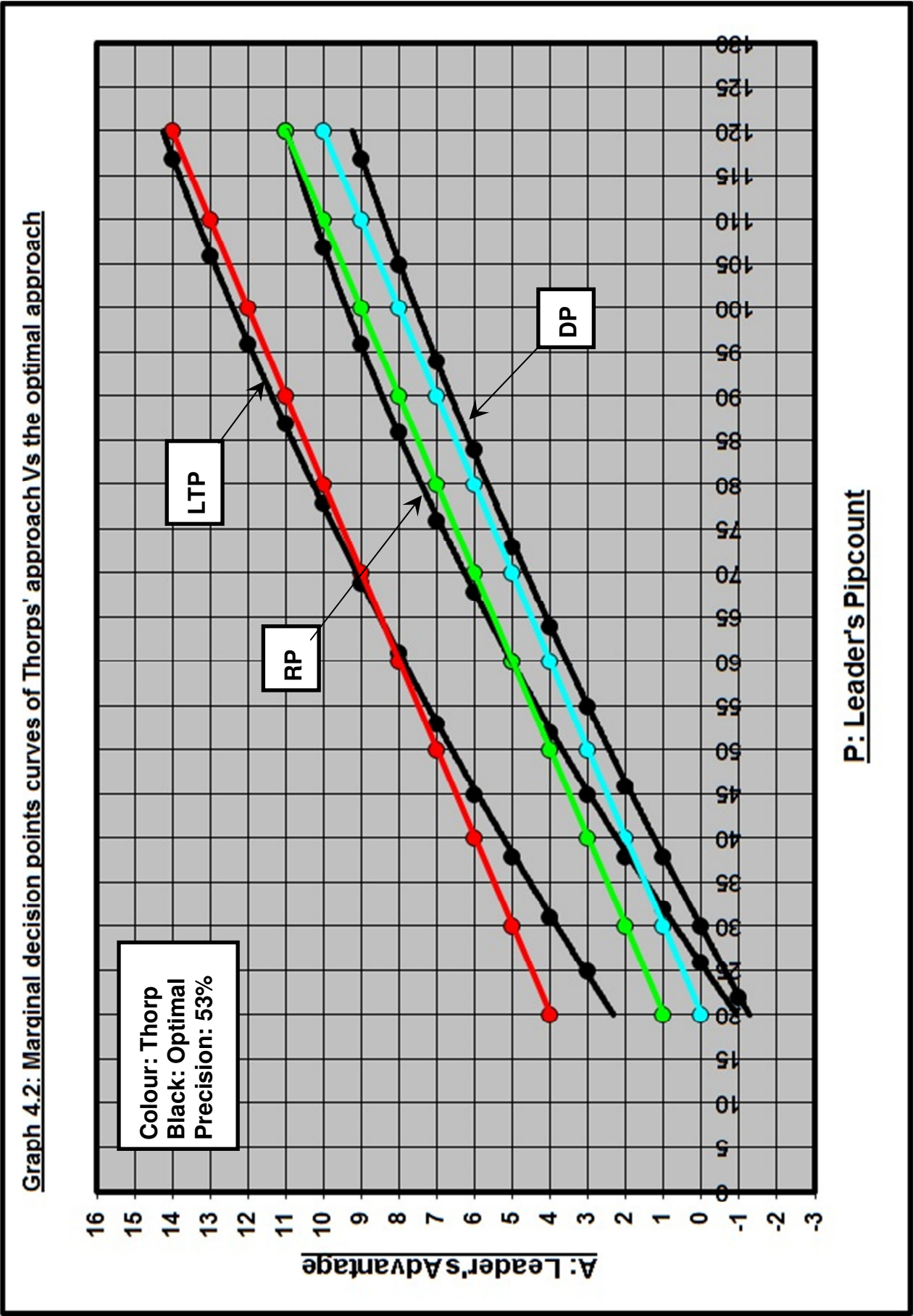
MDP	LTP
20	4
30	5
40	6
50	7
60	8
70	9
80	10
90	11
100	12
110	13
120	14

Table 4.3: Calculation of the precision of Thorp's approach

P	DP opt	DP obtained	GAP	RP opt	RP obtained	GAP	LTP opt	LTP obtained	Gap
20	-1	0	1	-1	1	2	2	4	2
21	-1	1	2	0	2	2	2	4	2
22	-1	1	2	0	2	2	2	4	2
23	0	1	1	0	2	2	2	4	2
24	0	1	1	0	2	2	2	4	2
25	0	1	1	0	2	2	3	4	1
26	0	1	1	0	2	2	3	4	1
27	0	1	1	1	2	1	3	4	1
28	0	1	1	1	2	1	3	4	1
29	0	1	1	1	2	1	3	4	1
30	0	1	1	1	2	1	3	5	2
31	1	2	1	1	3	2	4	5	1
32	1	2	1	1	3	2	4	5	1
33	1	2	1	2	3	1	4	5	1
34	1	2	1	2	3	1	4	5	1
35	1	2	1	2	3	1	4	5	1
36	1	2	1	2	3	1	4	5	1
37	1	2	1	2	3	1	4	5	1
38	1	2	1	2	3	1	5	5	0
39	2	2	0	3	3	0	5	5	0
40	2	2	0	3	3	0	5	5	0
41	2	3	1	3	4	1	5	6	1
42	2	3	1	3	4	1	5	6	1
43	2	3	1	3	4	1	5	6	1
44	2	3	1	3	4	1	5	6	1
45	2	3	1	3	4	1	6	6	0
46	2	3	1	4	4	0	6	6	0
47	3	3	0	4	4	0	6	6	0
48	3	3	0	4	4	0	6	6	0
49	3	3	0	4	4	0	6	6	0
50	3	3	0	4	4	0	6	7	1
51	3	4	1	4	5	1	6	7	1
52	3	4	1	4	5	1	6	7	1
53	3	4	1	5	5	0	7	7	0
54	3	4	1	5	5	0	7	7	0
55	3	4	1	5	5	0	7	7	0
56	4	4	0	5	5	0	7	7	0
57	4	4	0	5	5	0	7	7	0
58	4	4	0	5	5	0	7	7	0
59	4	4	0	5	5	0	7	7	0
60	4	4	0	5	5	0	7	8	1
61	4	5	1	6	6	0	8	8	0
62	4	5	1	6	6	0	8	8	0
63	4	5	1	6	6	0	8	8	0
64	4	5	1	6	6	0	8	8	0
65	5	5	0	6	6	0	8	8	0
66	5	5	0	6	6	0	8	8	0
67	5	5	0	6	6	0	8	8	0
68	5	5	0	6	6	0	8	8	0
69	5	5	0	7	6	1	9	8	1
70	5	5	0	7	6	1	9	9	0
71	5	6	1	7	7	0	9	9	0
72	5	6	1	7	7	0	9	9	0
73	5	6	1	7	7	0	9	9	0
74	6	6	0	7	7	0	9	9	0
75	6	6	0	7	7	0	9	9	0
76	6	6	0	7	7	0	9	9	0
77	6	6	0	8	7	1	9	9	0
78	6	6	0	8	7	1	10	9	1
79	6	6	0	8	7	1	10	9	1
80	6	6	0	8	7	1	10	10	0
81	6	7	1	8	8	0	10	10	0
82	6	7	1	8	8	0	10	10	0
83	6	7	1	8	8	0	10	10	0
84	6	7	1	8	8	0	10	10	0
85	7	7	0	8	8	0	10	10	0
86	7	7	0	8	8	0	10	10	0
87	7	7	0	9	8	1	11	10	1
88	7	7	0	9	8	1	11	10	1
89	7	7	0	9	8	1	11	10	1
90	7	7	0	9	8	1	11	11	0
91	7	8	1	9	9	0	11	11	0
92	7	8	1	9	9	0	11	11	0
93	7	8	1	9	9	0	11	11	0
94	7	8	1	9	9	0	11	11	0
95	8	8	0	9	9	0	11	11	0
96	8	8	0	9	9	0	11	11	0
97	8	8	0	10	9	1	12	11	1
98	8	8	0	10	9	1	12	11	1
99	8	8	0	10	9	1	12	11	1
100	8	8	0	10	9	1	12	12	0
101	8	9	1	10	10	0	12	12	0
102	8	9	1	10	10	0	12	12	0
103	8	9	1	10	10	0	12	12	0
104	8	9	1	10	10	0	12	12	0
105	8	9	1	10	10	0	12	12	0
106	9	9	0	10	10	0	13	12	1
107	9	9	0	10	10	0	13	12	1
108	9	9	0	11	10	1	13	12	1
109	9	9	0	11	10	1	13	12	1
110	9	9	0	11	10	1	13	13	0
111	9	10	1	11	11	0	13	13	0
112	9	10	1	11	11	0	13	13	0
113	9	10	1	11	11	0	13	13	0
114	9	10	1	11	11	0	13	13	0
115	9	10	1	11	11	0	13	13	0
116	9	10	1	11	11	0	13	13	0
117	9	10	1	11	11	0	14	13	1
118	10	10	0	11	11	0	14	13	1
119	10	10	0	11	11	0	14	13	1
120	10	10	0	11	11	0	14	14	0

Good results	159 on 303 = 52.5%
Results with a 1-pip difference	127 on 303 = 41.9%
Results with a 2-pip difference	17 on 303 = 5.6%
Results with a 3-pip or more difference	0 on 303 = 0.0%





2.5 Trice's approach

Trice's approach is a relatively recent one. This approach is presented in the book "Backgammon BOOT CAMP", in the chapter: "From Pipcount to Cube Action". This book was published in 2004 and the author is Mr. Walter Trice. Mr. Trice, who died in 2009, was considered one of the best backgammon players in the world. He wrote several articles about backgammon and he was among other things a special collaborator on the Internet site: "GammonVillage". His book, "Backgammon BOOT CAMP", is certainly one of the best book published so far. Reading this book is therefore strongly recommended.

The criteria of Trice's approach are:

When P is 62 pips or less

$$DP = (((P - 5)/7) - 3), \text{ down}$$

$$RP = (((P - 5)/7) - 2), \text{ down}$$

$$LTP = ((P - 5)/7), \text{ down}$$

When P is 63 pips or more

$$DP = ((P/10) - 2), \text{ up}$$

$$RP = ((P/10) - 1), \text{ up}$$

$$LTP = ((P/10) + 1), \text{ up}$$

Table 5.1 presents the obtained values.

Graph 5.1 illustrates the obtained values.

Table 5.2 presents the marginal decision points.

Using this approach, when "P" (the leader's pipcount) is 100 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 108 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 109 pips.
- The trailer must take if his pipcount is equal or inferior to 111 pips.

Using this approach, when "P" (the leader's pipcount) is 50 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 53 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 54 pips.
- The trailer must take if his pipcount is equal or inferior to 56 pips.

Table 5.3 presents the calculations of the precision. The results obtained are:

- Good results: 71,0%
- Results with a 1-pip difference: 29.0%
- Results with a 2-pip or more difference: 0.0%

Graph 5.2 illustrates the marginal decision points curves of this approach Vs the optimal approach. By comparing these two approaches, you can easily observe that the differences are relatively small.

With regards to all the analyzed approaches, this approach produces very good results. Consequently, this approach is a recommendable one.

With regards to all the analyzed approaches, this approach is very difficult to memorize.

* * * * *

We have seen in section 1.10, that Mr. Trice has also worked to develop a "theoretical" LTP marginal decision points curve. The LTP marginal decision points curve as obtained by Trice was illustrated in graph 12. As illustrated in graph 13, the LTP marginal decision points curve as obtained by Trice is almost identical to the LTP marginal decision points curve of the optimal approach. Please note that the "theoretical" marginal decision points as obtained by Trice form a curve.

In this present section, we have also seen that Trice also worked to develop a "practical" LTP marginal decision points curve, which was illustrated on graph 5.2. Please note that together, the "practical" LTP marginal decision points curve of Trice's approach consists of "two straight lines". Indeed, there is a "straight line" for values higher than 63 pips and another "straight line" for values below 62 pips.

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List of the tables and graphs of section 2.5

Table 5.1: Obtained values for Trice's approach

Table 5.2: Marginal Decision Points of Trice's approach

Table 5.3: Calculation of the precision of Trice's approach

Graph 5.1: Obtained values for Trice's approach

Graph 5.2: Marginal decision points curves of Trice's approach Vs the optimal approach

Table 5.1: Obtained values for Trice's approach

P	DP	RP	LTP
20	-1	0	2
21	-1	0	2
22	-1	0	2
23	-1	0	2
24	-1	0	2
25	-1	0	2
26	0	1	3
27	0	1	3
28	0	1	3
29	0	1	3
30	0	1	3
31	0	1	3
32	0	1	3
33	1	2	4
34	1	2	4
35	1	2	4
36	1	2	4
37	1	2	4
38	1	2	4
39	1	2	4
40	2	3	5
41	2	3	5
42	2	3	5
43	2	3	5
44	2	3	5
45	2	3	5
46	2	3	5
47	3	4	6
48	3	4	6
49	3	4	6
50	3	4	6
51	3	4	6
52	3	4	6
53	3	4	6
54	4	5	7
55	4	5	7
56	4	5	7
57	4	5	7
58	4	5	7
59	4	5	7
60	4	5	7
61	5	6	8
62	5	6	8
63	5	6	8
64	5	6	8
65	5	6	8
66	5	6	8
67	5	6	8
68	5	6	8
69	5	6	8
70	5	6	8

P	DP	RP	LTP
71	6	7	9
72	6	7	9
73	6	7	9
74	6	7	9
75	6	7	9
76	6	7	9
77	6	7	9
78	6	7	9
79	6	7	9
80	6	7	9
81	7	8	10
82	7	8	10
83	7	8	10
84	7	8	10
85	7	8	10
86	7	8	10
87	7	8	10
88	7	8	10
89	7	8	10
90	7	8	10
91	8	9	11
92	8	9	11
93	8	9	11
94	8	9	11
95	8	9	11
96	8	9	11
97	8	9	11
98	8	9	11
99	8	9	11
100	8	9	11
101	9	10	12
102	9	10	12
103	9	10	12
104	9	10	12
105	9	10	12
106	9	10	12
107	9	10	12
108	9	10	12
109	9	10	12
110	9	10	12
111	10	11	13
112	10	11	13
113	10	11	13
114	10	11	13
115	10	11	13
116	10	11	13
117	10	11	13
118	10	11	13
119	10	11	13
120	10	11	13

Table 5.2: Marginal Decision Points of Trice's approach

MDP	DP
25	-1
32	0
39	1
46	2
53	3
60	4
70	5
80	6
90	7
100	8
110	9
120	10

MDP	RP
25	0
32	1
39	2
46	3
53	4
60	5
70	6
80	7
90	8
100	9
110	10
120	11

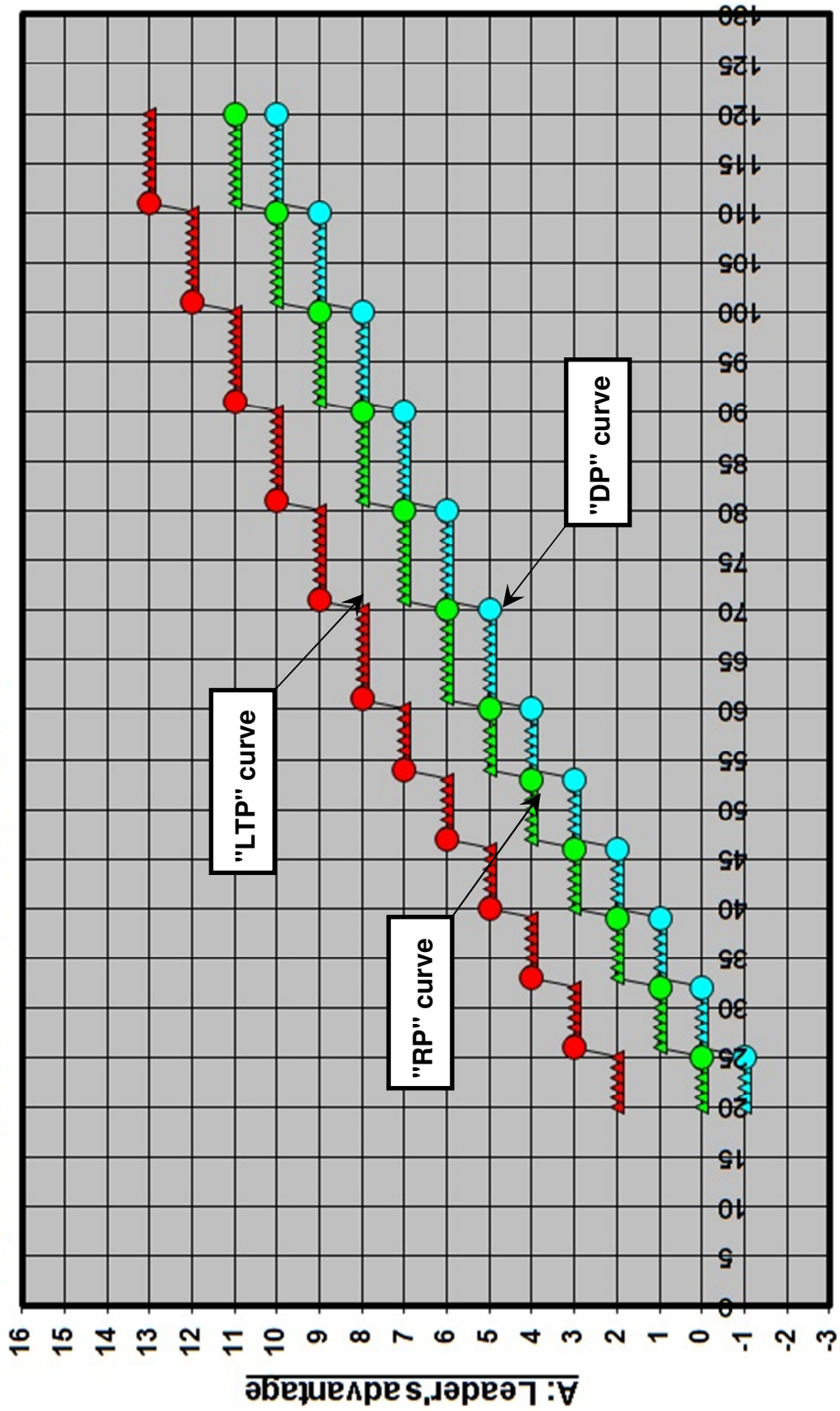
MDP	LTP
26	3
33	4
40	5
47	6
54	7
61	8
71	9
81	10
91	11
101	12
111	13

Table 5.3: Calculation of the precision of Trice's approach

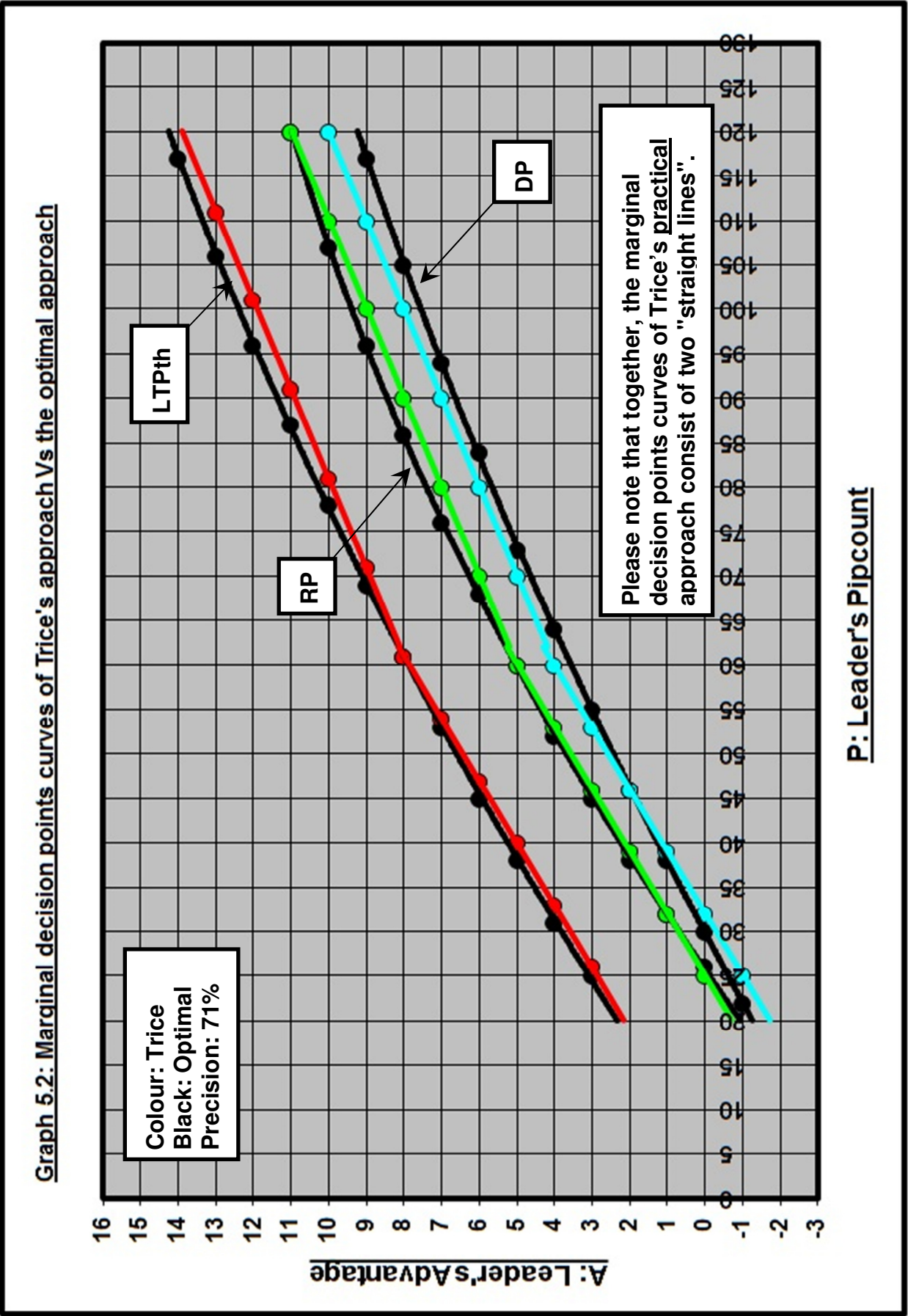
P	DP opt	DP obtained	Gap	RP opt	RP obtained	Gap	LTP opt	LTP obtained	Gap
20	-1	-1	0	-1	0	1	2	2	0
21	-1	-1	0	0	0	0	2	2	0
22	-1	-1	0	0	0	0	2	2	0
23	0	-1	1	0	0	0	2	2	0
24	0	-1	1	0	0	0	2	2	0
25	0	-1	1	0	0	0	3	2	1
26	0	0	0	0	1	1	3	3	0
27	0	0	0	1	1	0	3	3	0
28	0	0	0	1	1	0	3	3	0
29	0	0	0	1	1	0	3	3	0
30	0	0	0	1	1	0	3	3	0
31	1	0	1	1	1	0	4	3	1
32	1	0	1	1	1	0	4	3	1
33	1	1	0	2	2	0	4	4	0
34	1	1	0	2	2	0	4	4	0
35	1	1	0	2	2	0	4	4	0
36	1	1	0	2	2	0	4	4	0
37	1	1	0	2	2	0	4	4	0
38	1	1	0	2	2	0	5	4	1
39	2	1	1	3	2	1	5	4	1
40	2	2	0	3	3	0	5	5	0
41	2	2	0	3	3	0	5	5	0
42	2	2	0	3	3	0	5	5	0
43	2	2	0	3	3	0	5	5	0
44	2	2	0	3	3	0	5	5	0
45	2	2	0	3	3	0	6	5	1
46	2	2	0	4	3	1	6	5	1
47	3	3	0	4	4	0	6	6	0
48	3	3	0	4	4	0	6	6	0
49	3	3	0	4	4	0	6	6	0
50	3	3	0	4	4	0	6	6	0
51	3	3	0	4	4	0	6	6	0
52	3	3	0	4	4	0	6	6	0
53	3	3	0	5	4	1	7	6	1
54	3	4	1	5	5	0	7	7	0
55	3	4	1	5	5	0	7	7	0
56	4	4	0	5	5	0	7	7	0
57	4	4	0	5	5	0	7	7	0
58	4	4	0	5	5	0	7	7	0
59	4	4	0	5	5	0	7	7	0
60	4	4	0	5	5	0	7	7	0
61	4	5	1	6	6	0	8	8	0
62	4	5	1	6	6	0	8	8	0
63	4	5	1	6	6	0	8	8	0
64	4	5	1	6	6	0	8	8	0
65	5	5	0	6	6	0	8	8	0
66	5	5	0	6	6	0	8	8	0
67	5	5	0	6	6	0	8	8	0
68	5	5	0	6	6	0	8	8	0
69	5	5	0	7	6	1	9	8	1
70	5	5	0	7	6	1	9	8	1
71	5	6	1	7	7	0	9	9	0
72	5	6	1	7	7	0	9	9	0
73	5	6	1	7	7	0	9	9	0
74	6	6	0	7	7	0	9	9	0
75	6	6	0	7	7	0	9	9	0
76	6	6	0	7	7	0	9	9	0
77	6	6	0	8	7	1	9	9	0
78	6	6	0	8	7	1	10	9	1
79	6	6	0	8	7	1	10	9	1
80	6	6	0	8	7	1	10	9	1
81	6	7	1	8	8	0	10	10	0
82	6	7	1	8	8	0	10	10	0
83	6	7	1	8	8	0	10	10	0
84	6	7	1	8	8	0	10	10	0
85	7	7	0	8	8	0	10	10	0
86	7	7	0	8	8	0	10	10	0
87	7	7	0	9	8	1	11	10	1
88	7	7	0	9	8	1	11	10	1
89	7	7	0	9	8	1	11	10	1
90	7	7	0	9	8	1	11	10	1
91	7	8	1	9	9	0	11	11	0
92	7	8	1	9	9	0	11	11	0
93	7	8	1	9	9	0	11	11	0
94	7	8	1	9	9	0	11	11	0
95	8	8	0	9	9	0	11	11	0
96	8	8	0	9	9	0	12	11	1
97	8	8	0	10	9	1	12	11	1
98	8	8	0	10	9	1	12	11	1
99	8	8	0	10	9	1	12	11	1
100	8	8	0	10	9	1	12	11	1
101	8	9	1	10	10	0	12	12	0
102	8	9	1	10	10	0	12	12	0
103	8	9	1	10	10	0	12	12	0
104	8	9	1	10	10	0	12	12	0
105	8	9	1	10	10	0	12	12	0
106	9	9	0	10	10	0	13	12	1
107	9	9	0	10	10	0	13	12	1
108	9	9	0	11	10	1	13	12	1
109	9	9	0	11	10	1	13	12	1
110	9	9	0	11	10	1	13	12	1
111	9	10	1	11	11	0	13	13	0
112	9	10	1	11	11	0	13	13	0
113	9	10	1	11	11	0	13	13	0
114	9	10	1	11	11	0	13	13	0
115	9	10	1	11	11	0	13	13	0
116	9	10	1	11	11	0	13	13	0
117	9	10	1	11	11	0	14	13	1
118	10	10	0	11	11	0	14	13	1
119	10	10	0	11	11	0	14	13	1
120	10	10	0	11	11	0	14	13	1

Good results	215 on 303 = 71.0%
Results with a 1-pip difference	88 on 303 = 29.0%
Results with a 2-pip difference	0 on 303 = 0.0%

Graph 5.1: Obtained values for Trice's approach



P: Leader's Pipcount



2.6 Chabot's approach

Chabot's approach is a new approach. This approach was developed based on the marginal decision points curves of the optimal approach, which are illustrated in graph 2.2. To conceive this approach, the main objective was to find criteria very easy to remember and having a precision superior to the one obtained by using the 8%, 9%, 12% approach and using the Thorp's approach.

The criteria of this approach are:

$$DP = ((P/10) - 3), \text{ up}$$

$$RP = ((P/8) - 3), \text{ up}$$

$$LTP = P/8, \text{ down}$$

Table 6.1 presents the obtained values.

Graph 6.1 illustrates the obtained values.

Table 6.2 presents the marginal decision points.

Using this approach, when "P" (the leader's pipcount) is 100 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 107 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 110 pips.
- The trailer must take if his pipcount is equal or inferior to 112 pips.

Using this approach, when "P" (the leader's pipcount) is 50 pips, then:

- The leader must double if the trailer's pipcount is equal or superior to 52 pips.
- The leader must redouble if the trailer's pipcount is equal or superior to 54 pips.
- The leader must take if his pipcount is equal or inferior to 56 pips.

Table 6.3 presents the calculations of precision. The results obtained are:

- Good results: 63.0%
- Results with a 1-pip difference: 37.0%
- Results with a 2-pip or more difference: 0.0%

Graph 6.2 illustrates the marginal decision points curves of Chabot's approach Vs the optimal approach. By comparing these two approaches, we can easily observe that the differences are relatively small.

With regards to all the analyzed approaches, this approach produces fairly good results. Consequently, this approach is a recommendable one.

With regards to all analyzed approaches, this approach is very easy to remember.

Even if this approach is very easy to remember, to help you to memorize it, you could use the following "mnemonic tips":

- 1) When you have to decide to double or redouble, you do it to increase your gain; so you must round out the result obtained upward and you must use "up".
- 2) When you have to decide to take or pass, you do it to minimize your loses; so you must round out the result obtained downward and you must use "down".
- 3) When "P", the leader's Pipcount, is equal to 100 pips; the obtained results are: DP = 7 pips, RP = 10 pips, LTP = 12 pips. These results almost the same that the obtained results using the 8%, 9%, 12% approach giving the following results: DP = 8 pips, RP = 9 pips and LTP = 12 pips.

* * * * *

List of the tables and graphs of section 2.6

Table 6.1:	Obtained values for Chabot's approach
Table 6.2:	Marginal Decision Points of Chabot's approach
Table 6.3:	Calculation of the precision of Chabot's approach
Graph 6.1:	Obtained values for Chabot's approach
Graph 6.2:	Marginal decision points curves of Chabot's approach Vs the optimal approach

Table 6.1: Obtained values for Chabot's approach

P	DP	RP	LTP
20	-1	0	2
21	0	0	2
22	0	0	2
23	0	0	2
24	0	0	3
25	0	1	3
26	0	1	3
27	0	1	3
28	0	1	3
29	0	1	3
30	0	1	3
31	1	1	3
32	1	1	4
33	1	2	4
34	1	2	4
35	1	2	4
36	1	2	4
37	1	2	4
38	1	2	4
39	1	2	4
40	1	2	5
41	2	3	5
42	2	3	5
43	2	3	5
44	2	3	5
45	2	3	5
46	2	3	5
47	2	3	5
48	2	3	6
49	2	4	6
50	2	4	6
51	3	4	6
52	3	4	6
53	3	4	6
54	3	4	6
55	3	4	6
56	3	4	7
57	3	5	7
58	3	5	7
59	3	5	7
60	3	5	7
61	4	5	7
62	4	5	7
63	4	5	7
64	4	5	8
65	4	6	8
66	4	6	8
67	4	6	8
68	4	6	8
69	4	6	8
70	4	6	8

P	DP	RP	LTP
71	5	6	8
72	5	6	9
73	5	7	9
74	5	7	9
75	5	7	9
76	5	7	9
77	5	7	9
78	5	7	9
79	5	7	9
80	5	7	10
81	6	8	10
82	6	8	10
83	6	8	10
84	6	8	10
85	6	8	10
86	6	8	10
87	6	8	10
88	6	8	11
89	6	9	11
90	6	9	11
91	7	9	11
92	7	9	11
93	7	9	11
94	7	9	11
95	7	9	11
96	7	9	12
97	7	10	12
98	7	10	12
99	7	10	12
100	7	10	12
101	8	10	12
102	8	10	12
103	8	10	12
104	8	10	13
105	8	11	13
106	8	11	13
107	8	11	13
108	8	11	13
109	8	11	13
110	8	11	13
111	9	11	13
112	9	11	14
113	9	12	14
114	9	12	14
115	9	12	14
116	9	12	14
117	9	12	14
118	9	12	14
119	9	12	14
120	9	12	15

Table 6.2: Marginal Decision Points of Chabot's approach

MDP	DP
20	-1
30	0
40	1
50	2
60	3
70	4
80	5
90	6
100	7
110	8
120	9

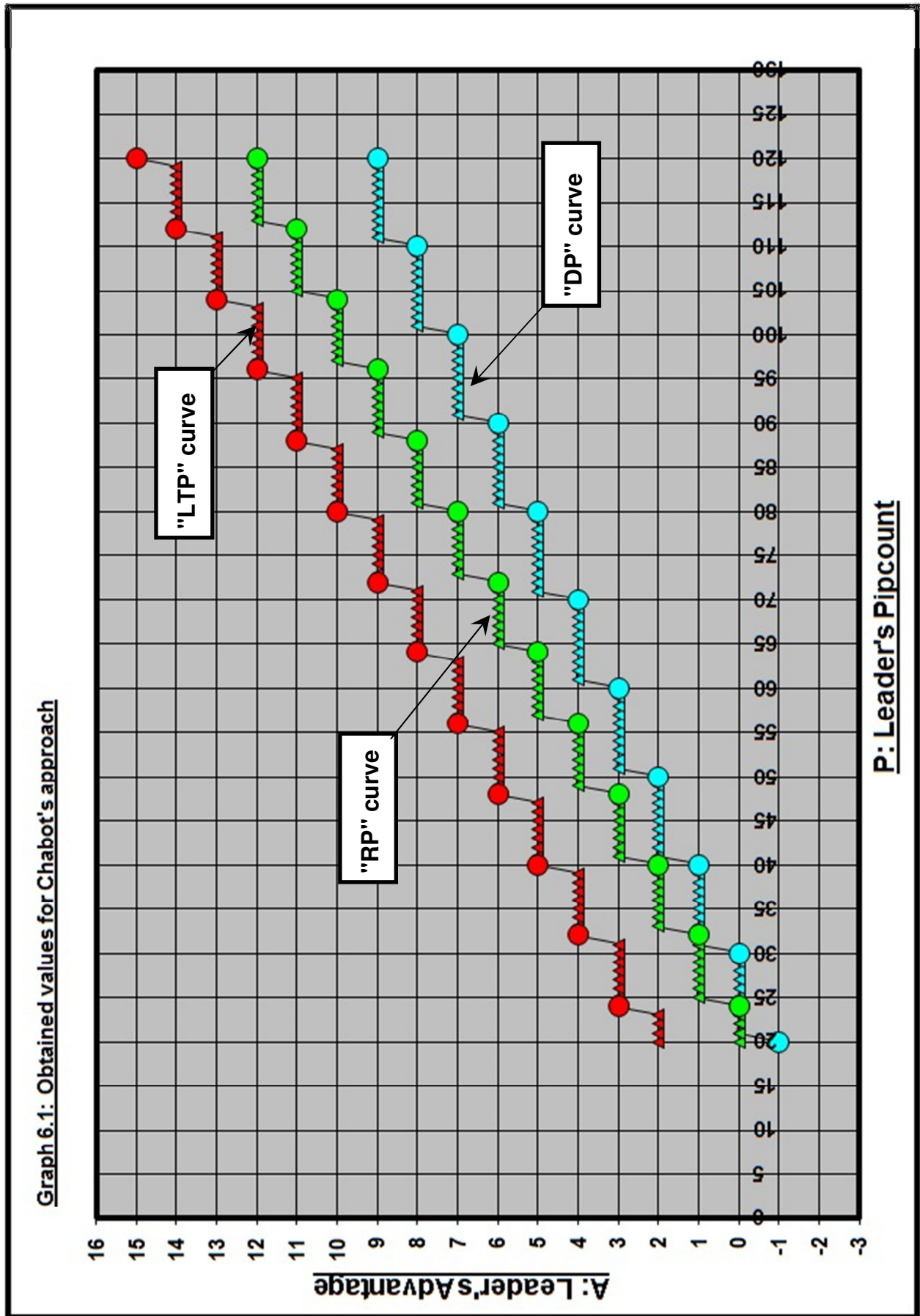
MDP	RP
24	0
32	1
40	2
48	3
56	4
64	5
72	6
80	7
88	8
96	9
104	10
112	11
120	12

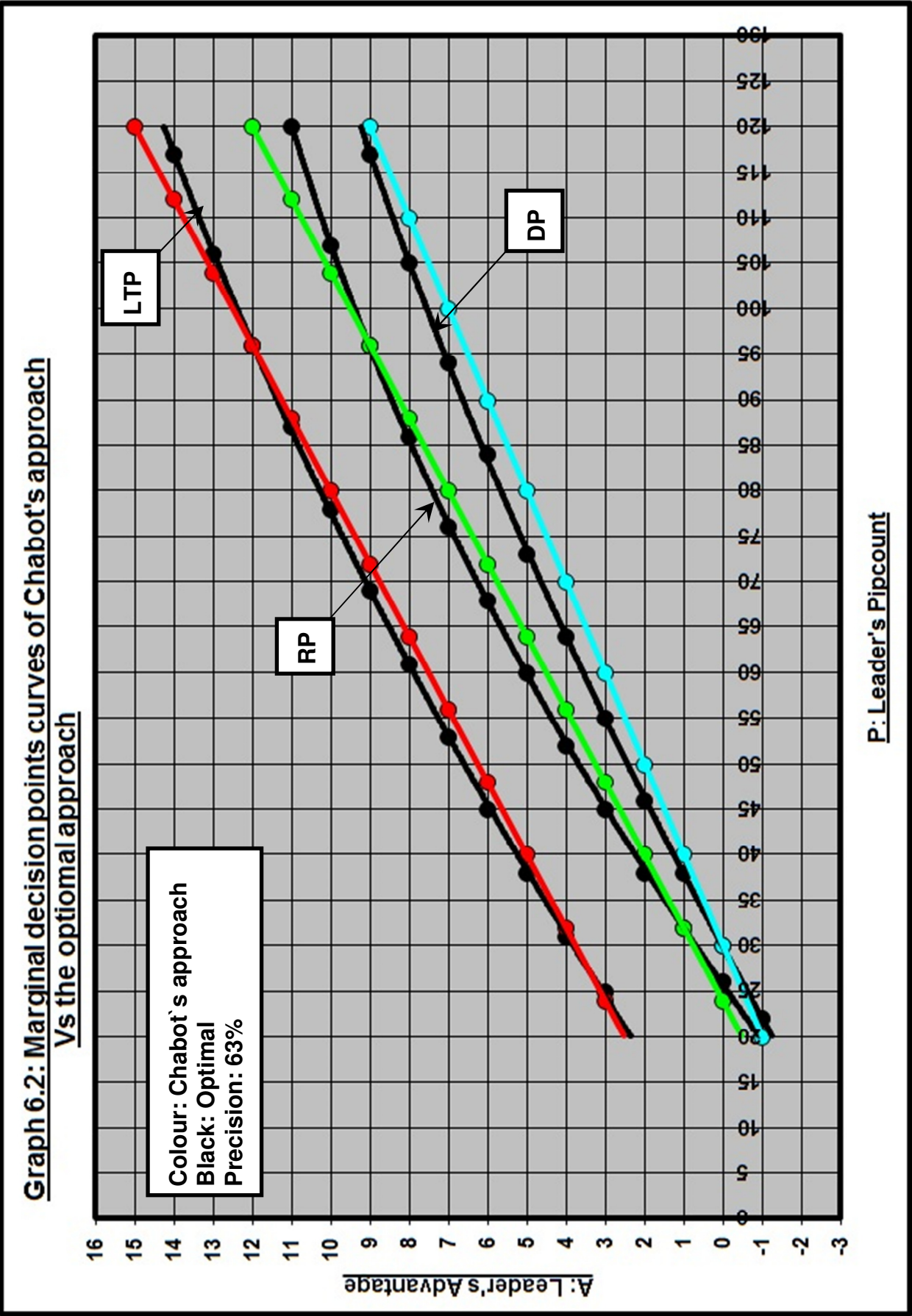
MDP	LTP
24	3
32	4
40	5
48	6
56	7
64	8
72	9
80	10
88	11
96	12
104	13
112	14
120	15

Table 6.3: Calculation of the precision of Chabot's approach

P	DP opt	DP obtained	Gap	RP opt	RP obtained	Gap	LTP opt	LTP obtained	Gap
20	-1	-1	0	-1	-1	0	2	2	0
21	-1	0	1	0	-1	1	2	2	0
22	-1	0	1	0	-1	1	2	2	0
23	0	0	0	0	-1	1	2	2	0
24	0	0	0	0	0	0	2	3	1
25	0	0	0	0	1	1	3	3	0
26	0	0	0	0	1	1	3	3	0
27	0	0	0	1	1	0	3	3	0
28	0	0	0	1	1	0	3	3	0
29	0	0	0	1	1	0	3	3	0
30	0	0	0	1	1	0	3	3	0
31	1	1	0	1	1	0	4	3	1
32	1	1	0	1	1	0	4	4	0
33	1	1	0	2	2	0	4	4	0
34	1	1	0	2	2	0	4	4	0
35	1	1	0	2	2	0	4	4	0
36	1	1	0	2	2	0	4	4	0
37	1	1	0	2	2	0	4	4	0
38	1	1	0	2	2	0	5	4	1
39	2	1	1	3	2	1	5	4	1
40	2	1	1	3	2	1	5	5	0
41	2	2	0	3	3	0	5	5	0
42	2	2	0	3	3	0	5	5	0
43	2	2	0	3	3	0	5	5	0
44	2	2	0	3	3	0	5	5	0
45	2	2	0	3	3	0	6	5	1
46	2	2	0	4	3	1	6	5	1
47	3	2	1	4	3	1	6	5	1
48	3	2	1	4	3	1	6	6	0
49	3	2	1	4	4	0	6	6	0
50	3	2	1	4	4	0	6	6	0
51	3	3	0	4	4	0	6	6	0
52	3	3	0	4	4	0	6	6	0
53	3	3	0	5	4	1	7	6	1
54	3	3	0	5	4	1	7	6	1
55	3	3	0	5	4	1	7	6	1
56	4	3	1	5	4	1	7	7	0
57	4	3	1	5	5	0	7	7	0
58	4	3	1	5	5	0	7	7	0
59	4	3	1	5	5	0	7	7	0
60	4	3	1	5	5	0	7	7	0
61	4	4	0	6	5	1	8	7	1
62	4	4	0	6	5	1	8	7	1
63	4	4	0	6	5	1	8	7	1
64	4	4	0	6	5	1	8	8	0
65	5	4	1	6	6	0	8	8	0
66	5	4	1	6	6	0	8	8	0
67	5	4	1	6	6	0	8	8	0
68	5	4	1	6	6	0	8	8	0
69	5	4	1	7	6	1	9	8	1
70	5	4	1	7	6	1	9	8	1
71	5	5	0	7	6	1	9	8	1
72	5	5	0	7	6	1	9	9	0
73	5	5	0	7	7	0	9	9	0
74	6	5	1	7	7	0	9	9	0
75	6	5	1	7	7	0	9	9	0
76	6	5	1	7	7	0	9	9	0
77	6	5	1	8	7	1	9	9	0
78	6	5	1	8	7	1	10	9	1
79	6	5	1	8	7	1	10	9	1
80	6	5	1	8	7	1	10	10	0
81	6	6	0	8	8	0	10	10	0
82	6	6	0	8	8	0	10	10	0
83	6	6	0	8	8	0	10	10	0
84	6	6	0	8	8	0	10	10	0
85	7	6	1	8	8	0	10	10	0
86	7	6	1	8	8	0	10	10	0
87	7	6	1	9	8	1	11	10	1
88	7	6	1	9	8	1	11	11	0
89	7	6	1	9	9	0	11	11	0
90	7	6	1	9	9	0	11	11	0
91	7	7	0	9	9	0	11	11	0
92	7	7	0	9	9	0	11	11	0
93	7	7	0	9	9	0	11	11	0
94	7	7	0	9	9	0	11	11	0
95	8	7	1	9	9	0	11	11	0
96	8	7	1	9	9	0	12	12	0
97	8	7	1	10	10	0	12	12	0
98	8	7	1	10	10	0	12	12	0
99	8	7	1	10	10	0	12	12	0
100	8	7	1	10	10	0	12	12	0
101	8	8	0	10	10	0	12	12	0
102	8	8	0	10	10	0	12	12	0
103	8	8	0	10	10	0	12	12	0
104	8	8	0	10	10	0	12	13	1
105	8	8	0	10	11	1	12	13	1
106	9	8	1	10	11	1	13	13	0
107	9	8	1	10	11	1	13	13	0
108	9	8	1	11	11	0	13	13	0
109	9	8	1	11	11	0	13	13	0
110	9	8	1	11	11	0	13	13	0
111	9	9	0	11	11	0	13	13	0
112	9	9	0	11	11	0	13	14	1
113	9	9	0	11	12	1	13	14	1
114	9	9	0	11	12	1	13	14	1
115	9	9	0	11	12	1	13	14	1
116	9	9	0	11	12	1	14	14	0
117	9	9	0	11	12	1	14	14	0
118	10	9	1	11	12	1	14	14	0
119	10	9	1	11	12	1	14	14	0
120	10	9	1	11	12	1	14	15	1

Good results	191 on 303 = 63.0%
Results with a 1-pip difference	112 on 303 = 37.0%
Results with a 2-pip difference	0 on 303 = 0.0%





2.7 Summary of the analyzed approaches

The analyzed approaches in part 2 of this article are:

- Optimal
- 8%, 9%, 12%
- Thorp
- Trice
- Chabot

Classified according to the obtained precision, we get:

Approach	Good results	Results with a 1-pip difference	Results with a 2-pip or more difference
8%, 9%, 12%	40.9%	41.6%	17.5%
Thorp	52.5%	41.9%	5.6%
Chabot	63.0%	37.0%	0.0%
Trice	71.0%	29.0%	0.0%
Optimal	100.0%	0.0%	0.0%

Graph 7.1 presented at the end of this section, compares the marginal decision points curves of each analyzed approach to the marginal decision points curves of the optimal approach. We can observe that the more the marginal decision points curves of an analyzed approach is near to the marginal decision points curves of the optimal approach, the higher the precision.

We have already concluded that with regards to all the analyzed approaches, the 8%, 9%, 12% approach and Thorp's approach do not produce good results and that therefore, those two approaches are not recommendable. Consequently, there are only 3 recommendable approaches.

The first recommendable approach is the optimal one. This approach gives the best theoretical results. The only practical way to use this approach is to memorize table 2.2 which presents the marginal decision points.

With regards to all recommendable approaches, this approach is difficult to remember.

The second recommendable approach is that of Trice, giving a precision of 71%.

With regards to all recommendable approaches, this approach is very difficult to remember.

The third recommendable approach is that of Chabot, giving a precision of 63%.

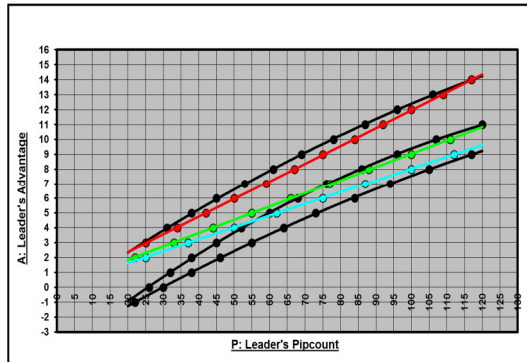
With regards to all recommendable approaches, this approach is very easy to remember.

The analyses carried out in part 2 of this article are summarized as follows:

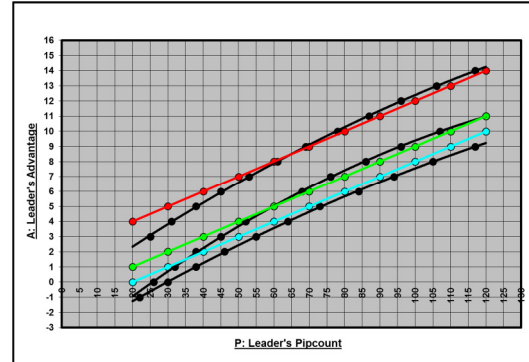
Approach	Precision	Remark	Easy to remember
8%, 9%, 12%	41%	Not recommendable	Not relevant
Thorp	53%	Not recommendable	Not relevant
Chabot	63%	Recommendable	Very easy
Trice	71%	Recommendable	Very difficult
Optimal	100%	Recommendable	Difficult

Graph 7.1: Marginal Decision Points curves of all analyzed approaches (in COLOR) Vs Marginal Decision Points curves of the optimal approach (in BLACK).

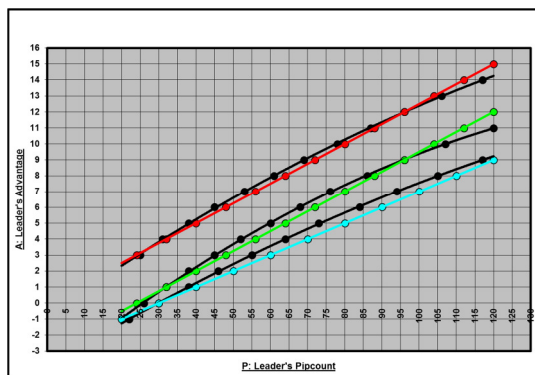
8%, 9%, 12 %
Precision: 41%
Not recommendable



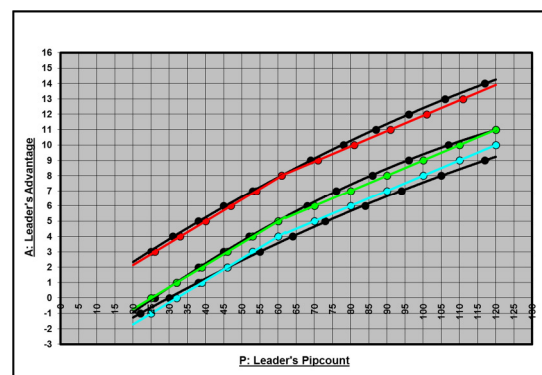
Thorp
Precision: 53%
Not recommendable



Chabot
Precision: 63%
Very easy to remember



Trice
Precision: 71%
Very difficult to remember



Part 3: From theory to practice

3.1 Practical examples

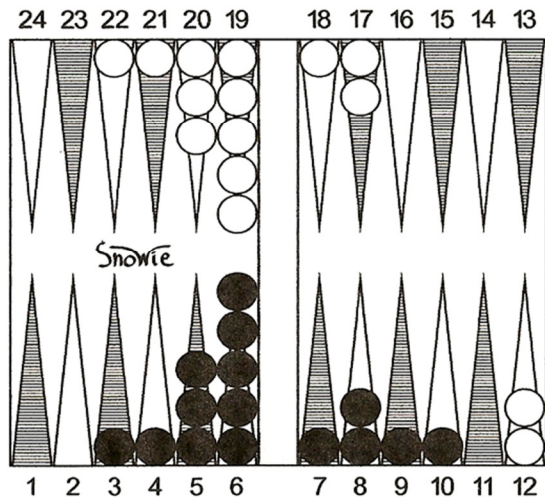
To illustrate in the most explicit way possible what must be done to correctly use an approach, we will analyze only 6 positions. The analyzed positions are "Low-wastage positions". These positions are obviously money game positions and for analysis purposes, the bet is set at \$10.00 a point.

The technique used to analyze every position is the following one:

- 1) Snowie was used to evaluate the equity of the analyzed position. The type of evaluation used is: "Full Cubeful, 3-Ply Play, 3-Ply Cube" so as to obtain a precision corresponds to the analysis of a minimum of 100,000 games played. Snowie's results are always expressed as "Normalized point per game".
- 2) For every analyzed approach, the criteria used are presented, the obtained results are presented and the financial loss, if there is one is presented.

The main goal of this section is not to analyze some specific positions but only to explain, in the most explicit way, how to correctly use the three recommendable approaches. We will also evaluate how easy to use an approach.

Position 1



pip:101

Bet: \$10.00/point**Black to play.**

1

pip:94

According to each of the 3 recommendable approaches, should Black double?

Expressed as "Normalized point per game", Snowie's results are:

Double, Take	0.672	
No double, Take	0.671	(-0.001)

Proper Cube Action: Double, Take

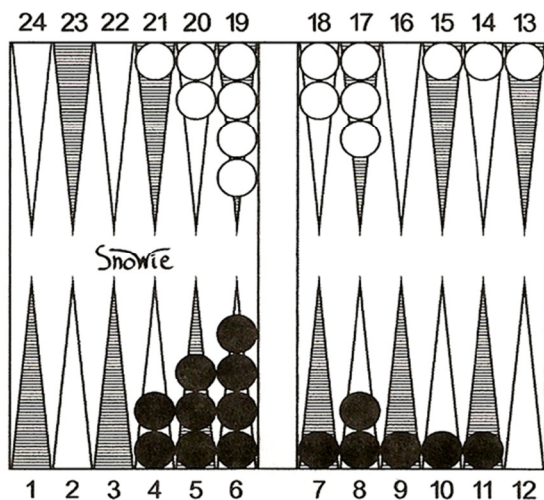
In this position, the leader's Pipcount is 94 pips ($P = 94$ pips) and the leader's Advantage is 7 pips ($A = 7$ pips). According to Snowie, Black should double.

If an approach gives a result of $DP = 7$ pips (or less), then this means that Black must double. Since the proper cube action is: "double", it's mean that doubling brings no error and no financial loss.

If an approach gives a result of $DP = 8$ pips (or more), then this means that Black must not double. Since the proper cube action is: "double", it's mean that not doubling is an error of 0.001 "Normalized point per game". Since the cube is at level 1, this corresponds to an error of 0.001 "Point per game" and brings a financial loss of \$0.01.

The obtained results are:

Approach used	Criterion	Result	Conclusion	Loss
Optimal	Table 2.2	7 pips	Double	\$0.00
Trice	$((P/10) - 2)$, up	8 pips	No double	\$0.01
Chabot	$((P/10) - 3)$, up	7 pips	Double	\$0.00

Position 2pip:1 0 9 **Bet: \$10.00/point****Black to play.**

According to each of the 3
 recommendable approaches, should
 Black redouble?

pip:1 0 0

2

Expressed as "Normalized point per game", Snowie's results are:

No redouble, Take	0.781	
Redouble, Take	0.777	(-0.004)

Proper Cube Action: No redouble, Take

In this position, the leader's Pipcount is 100 pips ($P = 100$ pips) and the leader's Advantage is 9 pips ($A = 9$ pips). According to Snowie, Black should not redouble.

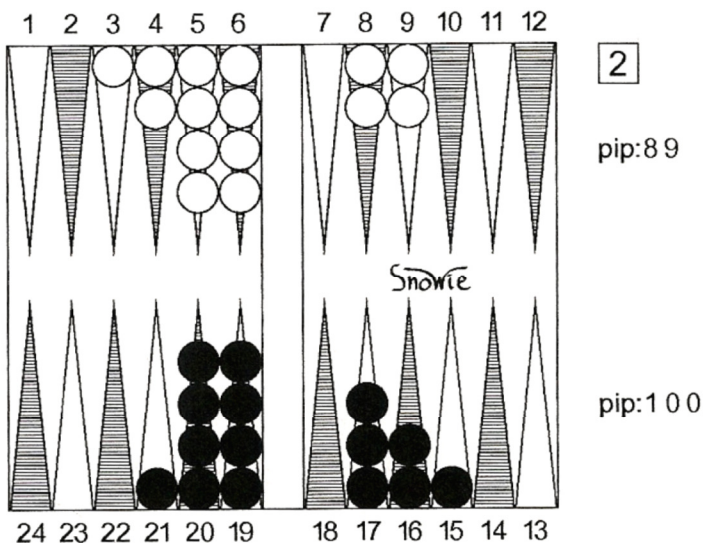
If an approach gives a result of $RP = 9$ pips (or less), then this means that Black must redouble. Since the proper cube action is: "no redouble", it's mean that redoubling is an error of 0.004 "Normalized point per game". Since the cube is at level 2, this corresponds to an error of 0.008 "Point per game" and brings a financial loss of \$0.08.

If an approach gives a result of $RP = 10$ pips (or more), then this means that Black must not redouble. Since the proper cube action is: "no redouble", it's mean that not redoubling brings no error and no financial loss.

The obtained results are:

Approach used	Criterion	Result	Conclusion	Loss
Optimal	Table 2.2	10 pips	No redouble	\$0.00
Trice	$((P/10) - 1)$, up	9 pips	Redouble	\$0.08
Chabot	$(P/8) - 3$, up	10 pips	No redouble	\$0.00

Position 3



Bet: \$10.00/point

White redoubles.

According to each of 3 recommendable approaches, should Black take or pass?

Expressed as "Normalized point per game", from Black's point of view, Snowie's results are:

Redouble, Take	-0.957	
Redouble, Pass	-1.000	(-0.043)

Proper Cube Action: Redouble, Take

In this position, the leader's Pipcount is 89 pips ($P = 89$ pips) and the leader's Advantage or the trailer's disadvantage is 11 pips ($A = D = 11$ pips). According to Snowie, Black should take.

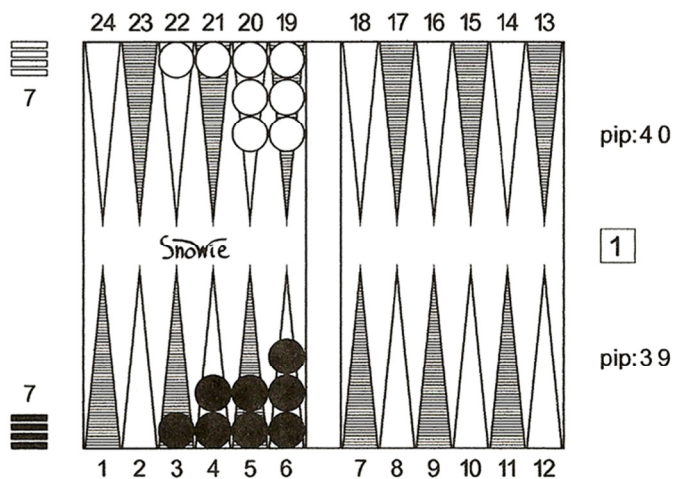
If an approach gives a result of $LTP = 10$ pips (or less), than this means that Black must Pass. Since the proper cube action is "take", passing is an error of 0.043 "Normalized point per game". Since the cube is at level 2, this corresponds to an error of 0.086 "Point per game" and being a financial loss of \$0.86.

If an approach gives a result of $LTP = 11$ pips (or more) than this means that Black must take. Since the proper cube action is "take", taking brings no error and no financial loss.

The obtained results are:

Approach used	Criterion	Result	Conclusion	Loss
Optimal	Table 2.2	11 pips	Take	\$0.00
Trice	$((P/10) + 1)$, up	10 pips	Pass	\$0.86
Chabot	$(P/8)$, down	11 pips	Take	\$0.00

Position 4



Bet: \$10.00/point

Black to play.

1

According to each of the 5 recommendable approaches, should Black double?

pip:40

pip:39

Expressed as "Normalized point per game", Snowie's results are:

No double, Take	0.489	
Double, Take	0.424	(-0.065)

Proper Cube Action: No Double, Take

In this position, the leader's Pipcount is 39 pips ($P = 39$ pips) and the leader's Advantage is 1 pip ($A = 1$ pip). According to Snowie, Black should not double.

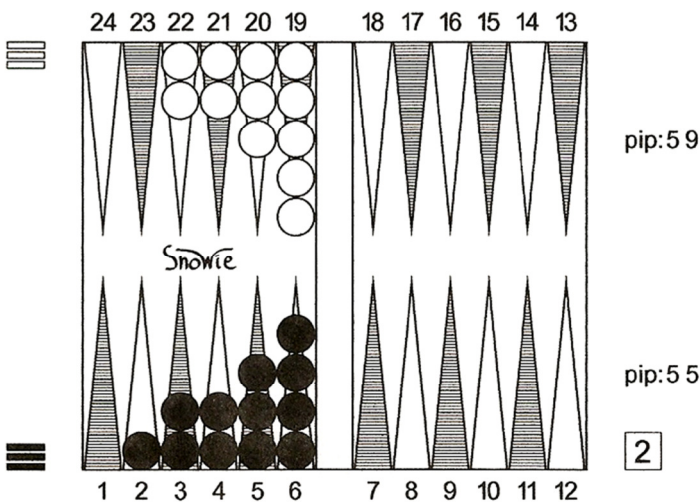
If an approach gives a result of $DP = 1$ pip (or less), then this means that Black must double. Since the proper cube action is "no double", doubling is an error of 0.065 "Normalized point per game". Since the cube is at level 1, this corresponds to an error of 0.065 "Point per game" and brings a financial loss of \$0.65.

If an approach gives a result of $DP = 2$ pips (or more), then this means that Black must not double. Since the proper cube action is "no double", not doubling brings no error and no financial loss.

The obtained results are:

Approach used	Criterion	Result	Conclusion	Loss
Optimal	Table 2.2	2 pips	No double	\$0.00
Trice	$((P - 5)/7) - 3$, down	1 pips	Double	\$0.65
Chabot	$((P/10) - 3)$, up	1 pips	Double	\$0.65

Position 5



Bet: \$10.00/point

Black to play.

According to each of the 3 the recommendable approaches, should Black redouble?

Expressed as "Normalized point per game", Snowie's results are:

No redouble, Take	0.690	
Redouble, Take	0.633	(-0.057)

Proper Cube Action: No redouble, Take

In this position, the leader's Pipcount is 55 pips ($P = 55$ pips) and the leader's Advantage is 4 pips ($A = 4$ pips). According to Snowie, Black should not redouble.

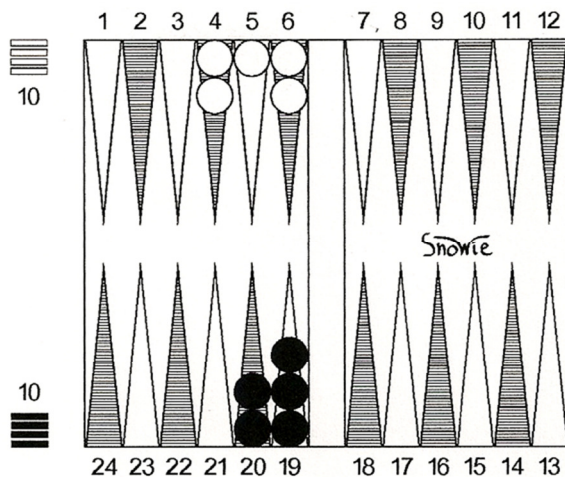
If an approach gives a result of $RP = 4$ pips (or less), then this means that Black must redouble. Since the proper cube action is "no redouble", redoubling is an error of 0.057 "Normalized point per game". Since the cube is at level 2, this corresponds to an error of 1.14 "Point per game" and brings a financial loss of \$1.14.

If an approach gives a result of $RP = 5$ pips (or more), then this means that Black must not redouble. Since the proper cube action is "no redouble", not redoubling brings no error and no financial loss.

The obtained results are:

Approach used	Criterion	Result	Conclusion	Loss
Optimal	Table 2.2	5 pips	No redouble	\$0.00
Trice	$((P - 5)/7) - 2$, down	5 pips	No redouble	\$0.00
Chabot	$(P/8) - 3$, up	4 pips	Redouble	\$1.14

Position 6



2

pip:25

Bet: \$10.00/point

White redoubles.

pip:28

According to each of the 3 recommendable approaches, should Black take or pass?

Expressed as "Normalized point per game", from Black point of view, Snowie's results are:

Redouble, Pass	-1.000	
Redouble, Take	-1.024	(-0.024)

Proper Cube Action: Redouble, Pass

In this position, the leader's Pipcount is 25 pips ($P = 25$ pips) and the leader's Advantage or the trailer's disadvantage is 3 pips ($A = D = 3$ pips). According to Snowie, Black should pass.

If an approach gives a result of $LTP = 2$ pips (or less), then this means that Black must "pass". Since the proper cube action is "pass", passing brings no error and no financial loss.

If the used approach gives a result of $LTP = 3$ pips (or more), then this means that Black must "take". Since the proper cube action is "pass", taking is an error of 0.024 "Normalized point per game". Since the cube is at level 2, this corresponds to an error of 0.048 "Point per game" and brings a financial loss of \$0.48.

The obtained results are:

Approach used	Criterion	Result	Conclusion	Loss
Optimal	Table 2.2	3 pips	Take	\$0.48
Trice	$((P - 5)/7)$, down	2 pips	Pass	\$0.00
Chabot	$(P/8)$, down	3 pips	Take	\$0.48

3.2 General observations

The obtained financial results for the 6 previous positions are summarized as follows:

Position	Optimal	Trice	Chabot
1	\$0.00	\$0.01	\$0.00
2	\$0.00	\$0.08	\$0.00
3	\$0.00	\$0.86	\$0.00
4	\$0.00	\$0.65	\$0.65
5	\$0.00	\$0.00	\$1.14
6	\$0.48	\$0.00	\$0.48
Total	\$0.48	\$1.60	\$2.27

As mentioned earlier, the main purpose of the analysis of those 6 positions was to illustrate how to use an approach in the most explicit way. The purpose of this analysis was not to demonstrate that one approach is better than another. Anyway, with the analysis of only 6 positions, it is obviously impossible to obtain any significant results and to give any valid conclusion.

Nevertheless, it is possible to make certain general observations, such as:

- 1) Among the 3 recommendable approaches, none produces perfect results.
- 2) Table 2.2 of the optimal approach is difficult to remember, but very easy to use.
- 3) Trice's approach is very difficult to remember and very difficult to use.
- 4) Chabot's approach is very easy to remember and very easy to use.

* * * * *

In summary, we have:

Approach	Easy to remember	Easy to use
Optimal	Difficult	Very easy
Trice	Very difficult	Very difficult
Chabot	Very easy	Very easy

3.3 Pipcount calculation

When you have tried to resolve the 6 previous problems, the pipcount had already been calculated by Snowie and the obtained pipcount had already been given in the presented "problem". You most likely did not verify it, but when you play, it is absolutely necessary that you correctly calculated the pipcount.

To correctly calculate a pipcount, you have to set a certain amount of time. And to correctly calculate a criterion which you retained, you also have to set a certain amount of time.

Do the following exercise:

- 1) Use Chabot's approach which is the easiest approach to remember and the easiest to use.
- 2) Redo the 6 previous problems.
- 3) Evaluate the approximate amount of time you will use to correctly calculate the pipcount.
- 4) Evaluate the approximate amount of time you will use to correctly calculate the criterion.

You will certainly notice that the amount of time necessary to correctly calculate the pipcount is approximately 10 times superior to the amount of time necessary to correctly calculate the criterion. And you will probably reach a similar result with whichever approach you use.

This last observation means that before correctly using any criterion, not only you must know how to correctly calculate the pipcount but you must really do it. This also means that, as a rule, you must correctly calculate the pipcount within a limited amount of time that you and your opponent will consider as being reasonable. For example, as an indication only, if in general, most of your opponents take about 20 seconds to calculate a pipcount, then you could consider this average as being a reasonable amount of time and you should consider this average as an acceptable standard.

Consequently, you have to find what should be your own "standard" and you should "work" to reach your own "standard". You should even try to improve your own "standard". To reach your own objective, you could consult certain internet sites to obtain suitable advices. To find several articles dealing with this specific subject and giving excellent advices, proceed as follows:

- 1) Go to the site: "Backgammon Galore!"
- 2) Choose the subject: "Articles By Topics"
- 3) In the topic: "4. Cube Handling", choose the subject: "Pip Counting"

If you participate in tournaments in which the use of a clock is imposed, then it is obviously in your best interest to be able to correctly calculate the pipcount in the shortest amount of time possible and you must train yourself to reach your own objectives.

CONCLUSION

The origin of this article was to find the answers the two following questions:

1) *Is it possible to evaluate the precision of an approach the objective of which is to obtain a proper "Money Cube Action in Low-Wastage Positions"?*

2) *If yes, what is the best approach?*

In part 1, we developed the optimal approach. The technique used to develop this approach was presented in a very detailed way to allow any skeptical reader to be able to really verify this technique, and to be able to confirm that the obtained approach is really the optimal one.

In part 2, we analyzed some approaches and we obtained the following results:

Approach	Precision	Remark	Easy to remember
8%, 9%, 12%	41%	Not recommendable	Not relevant
Thorp	53%	Not recommendable	Not relevant
Chabot	63%	Recommendable	Very easy
Trice	71%	Recommendable	Very difficult
Optimal	100%	Recommendable	Difficult

In part 3, we analyzed 6 positions, and we obtained the following results:

Approach	Easy to remember	Easy to use
Optimal	Difficult	Very easy
Trice	Very difficult	Very difficult
Chabot	Very easy	Very easy

Among the 3 recommendable approaches, the recommended approaches are the optimal one and the Chabot one. Trice's approach is not recommended, because this approach is too much difficult to remember and too much difficult to use.

The optimal approach is recommended, because this approach gives the best theoretical results, and because it is very easy to use. However, to use this approach, you must memorize table 2.2 presented below:

Table 2.2 Marginal Decision Points of the optimal approach

MDP	DP	MDP	RP	MDP	LTP
22	-1	20	-1	25	3
30	0	26	0	31	4
38	1	32	1	38	5
46	2	38	2	45	6
55	3	45	3	53	7
64	4	52	4	61	8
73	5	60	5	69	9
84	6	68	6	78	10
94	7	76	7	87	11
105	8	86	8	96	12
117	9	96	9	106	13
		107	10	117	14
		120	11		

Chabot's approach is recommended, because this approach gives fairly good results (better than 8%, 9%, 12% approach, better than Thorp's approach); and because this approach is very easy to remember, and very easy to use. To use this approach, you must use the following criteria:

DP (Doubling Point) = $((P/10) - 3)$, up

RP (Redoubling Point) = $((P/8) - 3)$, up

LTP (Last Take Point) = $P/8$, down

Here is the final conclusion:

To obtain a proper "Money Cube Action in Low-Wastage Positions"; use either the optimal approach or the Chabot one. Use the optimal approach, if you wish to memorize Table 2.2; otherwise, use the Chabot one.