

The Mathematics of Backgammon by Chuck Bower
The Thorp Count, Revised

Mathematician Edward O. Thorp is best known for his development of a blackjack system ${ }^{(1)}$ in the early 1960's which led the Las Vegas casinos to change the house rules for the popular game of 21 . Thorp also has made contributions to backgammon including a table for the correct handling of the doubling cube in all bearoffs where each player has exactly 2 checkers in his/her home board ${ }^{(2)}$. Bill Robertie in his classic Advanced Backgammon lists a formula, (coined the Thorp Count) for computing money game doubling strategy for noncontact races ${ }^{(3)}$. Though quite useful (and, in general, more accurate than other published race doubling formulas), the Thorp Count leaves room for improvement. With the aid of Expert Backgammon for the $P C^{(4)}$ software I have modified Thorp's method, and here give the Revised Thorp Count. I recommend this method for both money and match play when
a) no checkers remain outside of the home boards,
b) each player has at least four checkers remaining, and
c) at least one checker for each player is located higher than the three point.
(Note: Other methods work better when the three conditions above are not met.)

First I list the formula, and then give a couple of examples. If initially the formula appears complicated, don't despair. After you see the example positions worked through, it will probably seem much easier to apply. Then practice, practice, practice...

1) Calculate the roller's pip count, divide by 4 , round to the nearest whole number, and subtract the result from 74.
2) Take the difference in pip counts, multiply by 2 , and add to the result of step 1 if roller has fewer pips remaining; subtract if opponent has fewer.
3) Compare the number of points which remain covered in each home board. (Note: a point is covered if it has at least one checker on it.) Multiply the difference by 2 and add the result if roller has more points covered, subtract if opponent has more points covered.
4) If opponent has points with stacked checkers, add two for each checker on a point in excess of the
fourth. Do the same for the roller, except subtract instead of add.
5) Add four for each useless gap ${ }^{(5)}$ which the opponent has on his 3,4 and 5 points. Subtract four for each of roller's useless gaps on his/her 3, 4 and 5 points.
6) Compare the number of checkers each side has remaining. If both have the same number remaining, you are finished. If opponent has more checkers remaining, add to the result of step 5 for each excess checker as follows: 5 for the first, 10 for the second, 15 for the third, etc. If roller has more checkers remaining, then subtract instead of add.

The result of this calculation is the roller's winning chances (in percent) if no cube turns are allowed. A good rule of thumb for money play in non-contact races is: make an initial double with at least a $70 \%$ chance of winning if the game were rolled to completion (i.e. a cubeless game), redouble at $72 \%$ or higher, and drop if your opponent has better than a $78 \%$ chance in a cubeless rollout.
...continues page 8...
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Hoosier Pips: Larry Strommen placed second to Larry Taylor in the Atlanta (June) Monthly Backgammon Tournament...Thanks to Don Woods for directing Thursday evening backgammon while the Meeses' along with Woody Woodworth, Larry Strommen and David Smith took in the wonderful Michigan Summer Backgammon Championships in Novi, Michigan.


Coming Next Issue
FIBS?

Hoosier Backgammon Club's Newsletter for HBC members and subscribers.
Subscription rate: \$10/year (Canada \$12 and overseas \$14). Let us know if your address changes.
Butch \& Mary Ann Meese: (317) 845-8435. 7620 Kilmer Lane, Indianapolis, IN 46256-1634

Annotated match<br>Kit Woolsey vs Jeremy Bagai Internet-9 Point Match

In February, Kit Woolsey and Jeremy Bagai played a match and then annotated it for FIBS* players so they could see the thought process of the more experienced players. They played a fairly interesting match, logged it, and then annotated it independently. You will see reasons for their plays and cube decisions, as well as their second thoughts upon later analysis which often came to a different conclusion than their original choices.

Gerry Tesauro also volunteered TDGammon's valuable help. TD analyzed the whole match and listed its top 3 choices for each play along with its estimated equities. These equities are always assuming a 1 -cube and they do not take into account cube ownership. Thus on a pass-take decision an equity of -0.50 would be a break-even decision (not taking cube ownership into account -- that would probably make it a little higher), since that would translate to an equity of -0.100 on a 2 -
cube. TD was also nice enough to comment on the game, giving its reasons behind its choices as well as getting in a few snide remarks about their mistakes. Mark Damish (MA), first formatted the commentary for the Internet*.

Internet*: In short, the Internet is a network of computers. People login to an Internet server. Each server has a subset of features which may include email (electronic mail) and server-toserver connections. One of the servers provides a means for players to play each other - FIBS (First International Backgammon Server).

Editor's note: I felt that the material was too good to restrict it only to the Internet. I received permission from Kit, Jeremy, Gerry and Mark to reprint the match and I thank them. Those readers who are on the Internet can drop me a line and say hello (butch@inuxs.att.com).

In the backgammon positions, Kit is the black checkers and Jeremy the White. The board numbers are shown from the player on-roll point of view.

## Feedback from the MailBox

## Trust TD_Gammon??

You -are doing a good thing by printing the annotated match between Kit Woolsey and Jeremy Bagai. It is miraculous enough that TD-Gammon plays as well as reported. But to analyze a match and comment upon it in good colloquial English, often with sardonic humor? That's incredible, a much greater accomplishment even than programming first-rate backgammon.

When I see TD-Gammon ridiculing moves about which Kit and Jeremy concur, however, I wonder what's really going on. How does TD-Gammon estimate equities and how reliable are those estimates? And why should we believe TD-Gammon's analysis? Kit has been a top analyst for many years, and Jeremy appears to be very good also. Both have good (human) thought processes which usually lead to plays which, if not optimal, are very close to it.
...continues page 3...

## 1994 HOOSIER BACKGAMMON CLUB Gammon Point Standings. HBC Player of the Month for May is Cyrus Mobed with 218 gammon points. HBC, Player of the Month for June is Jim Curtis with 255 gammon points.

1) Chuck Stimming.......... 1248
2) Butch Meese............... 1000
3) Don Woods..................... 800
4) Ellis Bray........................ 616
5) Larry Strommen............ 586
6) Jim Curtis......................... 421
7) Cyrus Mobed.................... 412
8) Neil Ezell......................... 392
9) Jan Gurvitz................... 392
10) Woody Woodworth....... 380

Mary Ann Meese........... 370
Dragan Stevanovic....... 207

Kevin McLeaster. 172
Alan Haas...................... 160
Mick Dobratz................... 147
Bill Julian........................ 105
Rick Reahard................... 90
Chuck Bower................... 88
Dave Cardwell.................. 84
Scott Richardson............ 77
Craig Hampton................. 68
Stu Sherman.................... 60
Gabe Stiasny.................. 60
Steve Perlman................. 50

Frank Scott......................... 32 32
Wendy Kaplan................... 30
Jeff Baker............................ 20
John O'Hagan..................... 20
J.A. Miller............................ 20

Lance Jenkins..................... 20
Angie Jones........................ 10
Bill Gheen........................... 10
Sean Garber...................... 10
Krystal Shaffer.................... 10

|  | May 3rd |
| ---: | :--- |
| 1st | Chuck Stimming |
| 2nd | Butch Meese |
| 2nd | Cyrus Mobed |

May 10th
Cyrus Mobed Chuck Stimming Neil Ezell

May 17th
Mick Dobratz
Butch Meese
Jan Gurvitz

## May 24th <br> Don Woods <br> Dave Cardwell <br> Cyrus Mobed

June 23rd Chuck Strimming Woody Woodworth Bill Julian

June 30th
Jim Curtis Chuck Stimming Chuck Bower

## BACKGAMMON Tournament Schedule

July 29-31....42nd INDIANA Open, Radisson Hotel, Indianapolis................................................................... (317) 845-8435
Aug 14-17.....Las Vegas Open Tournament, Stardust Hotel \& Casino, Las Vegas, NV.................................... (702) 893-6025
Sep 5-11...... World Cup IV and Eastern Open, Harvey's Addison Hotel, Dallas............................................. (301) 299-8264
Oct 5-9........ 4th Illinois State \& America Cup, Indian Lakes Resort, Bloomingdale, IL........................................(708) 945-7801
Thursdays 7:00 PM at SPATS (842-3465) Castleton Square between J.C.Penneys \& L.S.Ayres.
845-8435

Can we trust TD-Gammon when it rates the 4 -anchor so poorly on pages 6 and 7 of the May-June 1994 HBC Newsletter? According to TDGammon, Jeremy's play of the 4-3 from the bar is wrong by 0.071, and both Kit and Jeremy are confused. Likewise, Jeremy's subsequent 4-3 is wrong by 0.038 , even though both Kit and Jeremy think the play too automatic to merit any comment.

I'm afraid I have to stick with human intelligence. Or might TD-Gammon's intelligence be human after all? Early in the 19th century, I recall reading, someone named Maelzel toured with a chess automaton called the Turk. Eventually, it was discovered that under the Turk's turban lay a human chess master. Might Paul Magriel be hiding behind TD-Gammon? Otherwise why should we believe TD-Gammon over Chad and Jeremy, let alone Kit and Jeremy?

Yours, Danny Kleinman


Kit: Once I have made my 2-point Jeremy's back man isn't covering anything useful, so he is correct to spring it out. Before it was an asset sitting back there; now it is just a liability.

Jeremy: Correct. Once Kit has made his 2-point, the annoyance factor of the blot on his ace point goes down considerably. Instead, it becomes a target which Kit can attack. Notice how quickly the blot became irrelevant if you thought it was a factor when Kit doubled.

TD-Gammon: Right, but quite close. Hanging back still has some value. However, running does negate Kit's last play of making the 2 -point.

|  |
| :---: |
|  |  |
|  |  |

BLACK to play 61?


Moved: 13/6
Jeremy: No shots.
TD-Gammon: Safety first. Not close.

| $13 / 6 \ldots \ldots \ldots \ldots . .+0.484$ |
| :---: |
| $13 / 7,6 / 5 \ldots \ldots \ldots .+0.300$ |
| $13 / 7,8 / 7 \ldots \ldots \ldots .+0.294$ |



Kit: This is better than 13/6. Jeremy wants maximum builders to make the bar point.

TD-Gammon: I like $13 / 11,13 / 8$, which also spreads the builders out. However, I will admit that these kinds of technical plays are not the strongest part of my game. They generally aren't too important in the grand scheme of things. My real edge is in overall positional judgment decisions, where I have it all over humans because of my vast experience and ability to accurately and objectively weigh all the relevant factors.

| $13 / 11,13 / 8 \ldots \ldots \ldots \ldots . .0 .459$ |
| ---: |
| $13 / 8,11 / 9 \ldots \ldots \ldots \ldots . .0 .481$ |
| $13 / 6 \ldots \ldots \ldots \ldots .0 .489$ |



Moved: 13/8 6/3

Kit: This figures to be better than $13 / 5$. The spare on the 8 -point allows me to handle an awkward six. After 13/5, both 6-1 and 6-4 would leave a shot next roll.

Jeremy: 13/5 looks more natural, but leaves a shot if his next roll is 6-4 or 61, whereas Kit's actual play doesn't. That looks like the only difference so I think Kit's play is right, even though I know I would have played 13/5 without a thought. Perhaps I won't next time.

TD-Gammon: My evaluator says $13 / 5$ is the best. However, Kit's play might be right. The problem is that you humans want your answers quickly, so you don't give me time to examine all the possibilities thoroughly and you can't build fast enough computers. Consequently I am limited to looking ahead only one move (i.e. Jeremy's response in this case) and can't see what will happen after Kit's next roll so the 6-1 and 6-4 danger he speaks of escapes me. Therefore, I guess I'll just have to trust you guys in positions such as this one, although I am reluctant to trust human backgammon judgment very far. I guess it's just as well -- if I made the best move all the time you guys probably wouldn't play with me any more. Really, this sort of nitpicking technical analysis bores me anyway -let's get back to the more exciting positional judgment area.
$13 / 5 \ldots \ldots \ldots \ldots \ldots+0.506$
$13 / 8,6 / 3 \ldots \ldots \ldots+\ldots .466$
$8 / 5,8 / 3 \ldots \ldots \ldots \ldots+0.409$


Kit: Jeremy is trying to maximize his builders for the bar and two points. I think $13 / 8,9 / 6$ is a bit better. This locks up the 8-point, which is valuable, and bringing a spare to the 6 -point can't be all bad.

Jeremy: I can't slot the bar or two points, so I bring in another builder. Stop playing for money if you liked 21/13.

TD-Gammon: | agree with Jeremy -spread them out. However, once again
my answers here my be somewhat suspect.

| $13 / 10,9 / 4 \ldots \ldots \ldots \ldots-0.428$ |
| ---: |
| $13 / 5 \ldots \ldots \ldots \ldots \ldots \ldots-0.442$ |
| $9 / 4,8 / 5 \ldots \ldots \ldots \ldots$ |



Moved: 8/2 6/2
Jeremy: No shots.
TD-Gammon: Safe is safe. Not even close.

| $8 / 2,6 / 2 \ldots \ldots \ldots \ldots+0.338$ |
| ---: |
| $13 / 3 \ldots \ldots \ldots \ldots+0.150$ |
| $13 / 9,13 / 7 \ldots \ldots \ldots+0.020$ |

WHITE to play 61?


Moved: 9/2
Kit: Even though Jeremy might get a shot next roll, he is quite correct to slot the 2-point. He needs that point in all variations, and slotting it is the best way to make it. If he plays $9 / 3,8 / 7$ things could get awkward next roll.

Jeremy: One point which Kit has emphasized over and over in his wonderful Matchqiz is the need to slot points in positions like this instead of being afraid of the parlay: opponent rolls a number that leaves a blot, you roll a number to hit that blot, opponent rolls a number that comes in hitting your slotted point. This is a pretty remote parlay. In addition, if I don't start the 2-point now it may be hard to make later. Slotting and covering is by far the easiest way to make points. Note that making the bar point is a big blunder, allowing Kit to clear his midpoint safely if he rolls an ace.

TD-Gammon: Good thinking, guys.

Like I always said -- you can't make


Moved: 13/8 13/7
Kit: This is a classic pay me now or pay me later situation. I can play safe with $8 / 3,8 / 2$ and pray for rain next turn, or I can do what I did. It is usually correct to pay later, but there are exceptions and I think this is one of them. There are several relevant factors:

1) If I get away with my play I will be in great shape -- a clear favorite to not leave any more shots.
2) The pay later play is very dangerous. My position would be completely stripped, and it would be quite likely that I would have to leave far more than an 11 number shot in the next roll or two.
3) Right now Jeremy has a blot in his inner board, but he figures to cover it next roll and have a perfect board. Thus, getting hit will probably be more serious next roll. All these factors are important. If one or more of them didn't exist paying later would probably be better, but here it looks right to shoot it out now.

Jeremy: Another issue that Kit analyzes often in Matchqiz is the classic "Pay me now or pay me later" question. The factors are: Is my opponent's board improving or crashing? If I pay later will I be likely to leave more and worse shots? If I pay now and am missed will I be home free. In this case the answers are: improving, much worse, and very nearly. Kit should pay now, and he does. If this sounds an awful lot like what Kit says it's because I study his Matchqiz almost daily. In the last 33 moves I have had occasion to draw upon the three backgammon sources that have influenced me the most. In their order of complexity, Magriel, Robertie, Woolsey: these three alone will have you winning tournaments. Of course if you really
want to be good, read everything, take lessons from pro's, and quit your job.

TD-Gammon: Now that's more like it. Enough of the technical stuff -- back to the real he-computer decisions which win or lose games. Pay me now or pay me later decisions are right up my alley. In this case, paying now was right by a country mile, for all the reasons the boys said. Well done, guys. You're finally starting to get your priorities straight.


Kit: Covering the blot is correct. There is virtually no gammon danger, and if Jeremy hits a shot he will definitely want to have his 2-point.

TD-Gammon: Correct. No reason not to cover the blot.

| $8 / 2 \ldots \ldots \ldots \ldots \ldots \ldots-0.622$ |
| ---: |
| $13 / 11,8 / 4 \ldots \ldots \ldots \ldots .645$ |
| $13 / 7 \ldots \ldots \ldots \ldots .0 .658$ |

BLACK to play 21?


Moved: 8/7 3/1
Kit: I'm not so sure about this play. It is clearly safer than the alternative of $8 / 6$, 7/6 for next roll, but it does leave a stripped position and two points to clear. However the fact that it is $100 \%$ safe next turn and doesn't figure to be so bad after that makes it look like the winner.

Jeremy: Hmmm. At first glance $8 / 6$, 7/6 appears to be better because it
clears a point rather than creating one However that play leaves a shot on 6-1 and 6-4 (once again) whereas this play doesn't.

TD-Gammon: Yeah, I know. My algorithm gives $7 / 5,2 / 1$ as best, and you guys laugh at this play. Well you can look ahead in this position better than I can, so I'll bow down and accept your judgment. I accomplished the important part -- clearing the midpoint safely -- now you guys can figure out how to lock up the win.

| $7 / 5,2 / 1 \ldots \ldots \ldots \ldots+0.696$ |
| :--- |
| $8 / 6,7 / 6 \ldots \ldots \ldots+0.680$ |
| $8 / 7,3 / 1 \ldots \ldots \ldots+0.671$ |

WHITE to play 44 ?


Moved: 13/1 5/1
BLACK to play 53 ?


Moved: 8/3 8/5.
WHITE played 21 by moving $13 / 10$. BLACK played 43 by moving 6/2 6/3. WHITE played 32 by moving $10 / 5$.

BLACK to play 44 ?


Moved: 7/3 7/3 5/1 5/1
Jeremy: Obviously better than 7/3, $5 / 1$ (3) in that it leaves 11 shots instead of 14.


Moved: 21/9
Kit: This is not right. The key is that I have 15 checkers left, so taking only one checker off is quite sufficient. If I had 14 checkers left then this play might be correct because it would prevent me from taking two checkers off on some rolls. His play can be quite costly if I roll something like 6-1, hit and take the checker off, and he flunks for a while. He should get both checkers moving.

Jeremy: 21/18, 21/12 looks like a slightly more efficient use of the last three in that it gains an extra crossover. However, I think my play is better because it forces Kit to safety his blot with an ace, two, or three instead of taking a second checker off. Although, now that I look closely I see that Kit has an odd number of checkers to take off (15) and looks likely he'll not miss in the future so taking only one off now isn't any loss. Of course, he could roll four aces and miss, in which case the extra checker could matter. The plays are very close.

TD-Gammon: You guys argue this one out. It's too detailed for me.
$21 / 9 \ldots \ldots \ldots \ldots \ldots .0-0.936$
$21 / 12,5 / 2 \ldots \ldots \ldots .0 .949$
$21 / 12,21 / 18 \ldots \ldots . . .0 .952$

BLACK played 64 by moving 5/0 3/0.

WHITE to play 32?


Moved: 21/16
Kit: 21/19, 9/6 is slightly better technique for getting off the gammon, since that play brings one checker exactly to the 6point. In practice, it is very unlikely to matter.
off the gammon, which might be an issue if Kit rolls three consecutive doubles and I wrong by a thousandth of a point.

BLACK played 31 by moving 3/0 1/0. WHITE played 53 by moving 16/11 9/6.

Jeremy: It transposed anyway.
BLACK played 33 by moving 3/0(4).
WHITE played 63 by moving $11 / 53 / 0$.
BLACK played 61 by moving 2/0 1/0.
WHITE played 43 by moving $4 / 03 / 0$.
BLACK played 61 to win game.

| Game 2 |
| :---: |
| BLACK(2) WHITE (0) |

BLACK (Kit) to play 51?


Moved: 13/8 24/23
TD-Gammon: It's nice to see that you guys are finally learning to play the opening rolls correctly. I'm glad some of my teachings have sunk in.

| $24 / 23,13 / 8 \ldots \ldots \ldots+0.021$ |
| ---: |
| $13 / 8,6 / 5 \ldots \ldots \ldots \ldots .0 .006$ |
| $24 / 18 \ldots \ldots \ldots \ldots . .0 .023$ |

WHITE (Jeremy) to play 52 ?


Moved: 13/8 24/22
Kit: This play of an early $5-2$ is becoming more popular as players are understanding the importance of splitting the back checkers quickly. Here it is probably best, since 13/11, 13/8 exposes the blot to two indirect shots.

Jeremy: It looked like this was better than $13 / 11,13 / 8$ because of the six fly shots, but those are balanced by the value of the builder when it is missed. The plays are probably equally good.

TD-Gammon: Correct. The split is important, and the indirect shots are too
costly.

| $24 / 22,13 / 8 \ldots \ldots \ldots . .-0.079$ |
| ---: |
| $13 / 11,13 / 8 \ldots \ldots \ldots .0 .098$ |
| $13 / 8,6 / 4 \ldots \ldots \ldots .0 .126$ |

BLACK (Kit) to play 65?


Moved: 24/18 23/18
Jeremy: Anchoring is certainly better than escaping one checker.

TD-Gammon: Right. Making the anchor is clear.

| $24 / 18,23 / 18 \ldots \ldots \ldots+0.125$ |
| ---: |
| $24 / 13 \ldots \ldots \ldots \ldots+0.033$ |
| $24 / 18,13 / 8 \ldots \ldots \ldots .+0.015$ |

WHITE (Jeremy) to play 11?


Moved: 22/20 6/5 6/5
Kit: Jeremy has several choices. The best defensive play is $24 / 21,22 / 21$, which gives him an advanced anchor and we would then be in a mutual holding game. Making the defensive three point with $24 / 22,6 / 5(2)$ is not as strong. Jeremy's play balances a strong offense with excellent board coverage. Even though he doesn't make an anchor, I will have a difficult time playing safely. I like his play.

Jeremy: The five point is a must. Anchoring on the 21 -point would be a waste considering that Kit is not yet threatening anything offensively. Advancing to the 20 -point covers more of the outtield in order to make it more difficult for Kit to bring builders down. Notice that even as early as my second roll all of Magriel's criteria argue for a bold play rather than a defensive play.

TD-Gammon: I'm not exactly sure why | like 22/21, 6/5(3) a bit better than Jeremy's play -- maybe because Jeremy is behind in the race preparing
to make the anchor a bit farther back and keeping more contact while not making Kit's aces strong is the idea. At any rate, Jeremy's play is fine, certainly better than making either of the anchors.

| $22 / 21,6 / 5(3) \ldots \ldots \ldots . .-0.092$ |
| ---: |
| $22 / 20,6 / 5(2) \ldots \ldots \ldots . .0 .098$ |
| $6 / 5(2), 6 / 4 \ldots \ldots \ldots .0 .099$ |

BLACK (Kit) to play 32?


Moved: 13/11 8/5x
Kit: I couldn't stomach the thought of playing safe with $13 / 8$, putting a fifth checker on the eight point, so I hit loose and went after my five point. I hate to say it, but I think the safe $13 / 8$ might be better because Jeremy has the stronger board but I am ahead in the race, therefore I don't want a blot hitting contest.

Jeremy: $13 / 8$ would be a mistake. My board is better, but it is only two points and Kit does have an anchor. The opening is a battle for the five points, and Kit rightly fights for his.

TD-Gammon: Kit shows excellent insight in his analysis -- too bad he couldn't find it at the table. $13 / 8$ is much better than hitting on the five point. The position thematically screams for a safe play. Kit is ahead in the race and has nobody stuck back, but Jeremy has the stronger board. Kit does not want to get involved in a blot hitting contest. He shouldn't even particularly care if Jeremy makes his five point.

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13/8
``` \(\qquad\)
``` \(+0.082\)
13/11, 8/5x
x.... . 8/5x/3 -0.014 8/5x/3 ................ -0.038
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WHITE (Jeremy) to play 42?


Moved: B/23 24/20x

BLACK (Kit) to play 32?


Moved: B/23 8/5x
Kit: I keep flailing away, but this strips my eight point as well as leaving many shots. I think I should have played the more solid $B / 22,13 / 11$ and waited for a better chance to attack.

Jeremy: I'm pretty sure this is right, although it's less clear than last time because Kit strips his 8 -point. An interesting example of when Magriel's criteria conflict with each other. Kit has more checkers back and an anchor which argue for bold plays, but has the weaker board which argues for safe plays.

TD-Gammon: Once again Kit's postmortem is better than his actual play. While I am a big fan of knocking my opponent off my 5-point, the circumstances have to be right for it. They just aren't here. Stripping the 8point and failing to lock up the 11-point cost too much. Hitting loose is a serious error.

| $B / 22,13 / 11 \ldots \ldots \ldots \ldots-0.102$ |
| ---: |
| $B / 23,11 / 8 \ldots \ldots \ldots \ldots \ldots .174$ |
| $B / 23,8 / 5 x \ldots \ldots \ldots . .0 .203$ |

WHITE (Jeremy) to play 41?


Moved: B/20x
BLACK (Kit) to play 61?


Moved: B/24 11/5x

Kit: Since I will have to leave a shot whatever I do, I might as well make one more try to win the fight for my five point.

Jeremy: This is clear because $B / 18$ leaves the direct shot anyway.

TD-Gammon: Correct, for all the
reasons given by the players.


Moved: $\mathrm{B} / 20 \times 8 / 6$
Kit: This is probably a bit better than $B / 20 x, 23 / 21$, since the balance of four checkers on the six point and three on the eight point is better than vice versa. The reason is that you are sometimes willing to give up your eight point in order to make an inner board point, but you will never want to give up your six point.

Jeremy: Four reasonable deuces: $23 / 21 ; 20 / 18 ; 13 / 11 ; 8 / 6$ ( $6 / 4$ is not reasonable - it leaves a triple direct shot and strips the 6 -point). $13 / 11$ is, in fact worthless because my 5 -point is made and my bar point is taken. The blot would simply be a target. 23/21 would be disadvantageous because it gives up coverage of Kit's inner board, allowing him to play safe behind me. $20 / 18$ is pretty neutral. $8 / 6$ is actually
beneficial. It's usually right to have more spares on the 6 -point than on the 8 -point. I'll be willing to break my 8point long before I break my 6-point.

TD-Gammon: Close, but Jeremy's accurate judgment is correct. The spare on the six point is better than on the eight point.
$B / 20 x, 8 / 6 \ldots \ldots \ldots \ldots+0.355$
$B / 20 x, 23 / 21 \ldots \ldots \ldots+0.341$
$B / 20 x, 13 / 11 \ldots \ldots \ldots+0.332$

BLACK (Kit) to play 54?


Moved: B/21 13/8
Kit: My play maximizes my chances to make another anchor, but I expose myself to a possible attack. The play is ok, but now I slightly prefer $\mathrm{B} / 21,23 / 18$. The third checker on Jeremy's bar point gives me some badly needed flexibility.

Jeremy: $23 / 18,21 / 16$, and $13 / 8$ are all reasonable 5's. I have no idea which is better but would probably make Kit's play.

TD-Gammon: As Jeremy says, all the fives are reasonable. However, bringing the spare to the bar point, which Kit suggests in his commentary, is the slight favorite. Flexibility is the key here; the attack forces can wait.


WHITE (Jeremy) to play 61?


Moved: 20/13
Kit: Jeremy is content to leave me alone and run one checker to safety. The problem with this play is that it is now too easy for me to make another anchor, after which my defense will be solid. I think he should have tried to hit me when I am down and back, but if I should happen to roll badly from the bar he would have excellent attacking chances.

Jeremy: Again Magriel's criteria point in different directions - I have the stronger board, but no anchor and fewer checkers back. Putting two in the air with $8 / 2 x / 1 x$ might lead to a quick blitz if Kit dances, but it leaves a direct shot, starts a point I don't really want, and strips my eight point. 20/13, disengaging, is better.

TD-Gammon: I hate it when such widely differing plays come out so close -- it makes me wonder if even I can play perfect backgammon. Jeremy's play won, but it was too close to be sure. But, don't ignore the much more flexible $20 / 14,6 / 5$. That prettier play goes well with my circuits, and almost won top billing.

| $20 / 13 \ldots \ldots \ldots \ldots .+0.316$ |
| ---: |
| $20 / 14,6 / 5 \ldots \ldots \ldots+0.312$ |
| $8 / 2 x / 1 x \ldots \ldots \ldots+0.310$ |

...continues next issue..

## The Thorp Count, Revised

 ...continues from Page 1...Now we are ready for an example. Position 1 is problem 83 from Inside Backgammon ${ }^{(6)}$. Should black redouble in a money game? Here we go!


1) Black's pip count is 44 , so dividing by 4 gives 11 ; subtracting this from 74 leaves 63.
2) Compare pips: If we (mentally) move one of black's checkers from his five point to his two point, then both players will have the same position. Thus black is behind by three pips so subtract $6(=2 \times 3)$ from 63 leaving 57 .
3) White has one more point covered (he has a checker on the two point but the roller doesn't), so subtract 2, leaving 55.
4) Neither player has a stack higher than three on any point, and since you only adjust the total when a point has more than four, proceed to step 5.
5) Neither player has a gap on the 3,4 , and 5 points (let alone a useless gap on these points) so no adjustment here.
6) Both players have the same number of checkers remaining. No further adjustments.
We can now conclude that black, on roll, has a $55 \%$ chance of winning if the game is rolled out to completion. This is certainly NOT a redouble. In fact, white should probably beaver if black were to mistakenly redouble! (Note that if white holds the cube he has higher than $45 \%$ winning chances, since he can use the cube to end some games which he otherwise would have lost. Ownership of the cube makes white the favorite, thus justifying his beaver.) A rollout by Expert Backgammon predicts that black has a $54 \%$ chance in a cubeless game. Thus our calculation appears accurate.

Position 2


Position 2 is from the 1981 Las Vegas Holiday Tournament ${ }^{(7)}$, a 15 point match between Mike Maxikuli (black) and Kent Goulding (white). White leads 8-7 and black is on roll with a centered cube. What are the cube decisions? I calculate the match equity drop/take point for white to be $21 \%$ with equal combatants and a perfectly efficient cube. Taking cube inefficiency into account, 22-23\% seems more likely. Thus this is basically a money cube (which is almost always the case in a match for low level cubes with lots of points to go). Let's calculate black's winning chances: Roller's pip count is 32 so $74-8=66$; he trails by 6 pips $\Rightarrow 66-12=54$. Roller has one fewer point covered: $54-2=52$. Neither player has a stack higher than 4 (no adjustment). White has a useless gap on his 4 point: $54+4=58$. Black has one fewer checker to bear off: $58+5=63$. So we conclude that black is a $63 \%$ favorite to win this game if the cube were banned from use. It appears that black should hold off turning the cube and white has a very easy take. A 10,000 trial rollout by Goulding showed black winning $62.6 \%$. In the actual match, Maxikuli doubled and Goulding took (and lost).

By now you may smell a rat. Did I hand pick positions which showed the Revised Thorp Count to be flawless? Maybe. But choosing an unbiased sample of 21 positions after having locked in the rules of the formula led to the following results: Formula correct within $2 \%$ : 12/21; formula correct to within 5\%: 18/21. Money cube decision predictions correct for both players: 19/21. I have one final warning: Rule \#6 (correcting for the advantage of having extra checkers already borne off) is highly volatile for differences of greater than one. The Revised Thorp Count needs more work on these (rare) positions.
(1) Edward O. Thorp, Beat the Dealer, Blaisdell Publishing Company (Random House), New York, 1962.
(2) Edward O. Thorp, The Mathematics of Gambling, Gambling Times, Inc. (Lyle Stewart, distr.), Secaucus NJ, 1984.
(3) Bill Robertie, Advanced Backgammon Vol. 2, The Gammon Press, Arlington MA, 1991, p. 191.
(4) Original version written by Tom Johnson, more recent versions (latest is 2.1) written by Tom Weaver of Dallas, TX. Interested readers should call Tom Weaver at (214) 692-1234. Product prices start at $\$ 50$.
(5) A gap is a point which has no checkers on it when checkers remain on higher points. A gap is useless if its corresponding number cannot fill a gap. For example, suppose you have checkers on your six point, but none on your five point. If you have an empty one point and you roll a five, you can fill the gap on the one point, so the gap on the five point is useful. If, on the other hand, you already have checkers on the one point, then you only succeed in stacking more there with the five, and thus your gap is useless.
(6) Kent Goulding and Bill Robertie (eds.), Inside Backgammon, vol. 4, no. 3, (May-June 1994) p. 23.
(7) Kent Goulding and Kit Woolsey, Backgammon with the Champions (edited by Joanne Goulding and Richard White), vol. 1, \#6, (March 1982) p. 32.

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