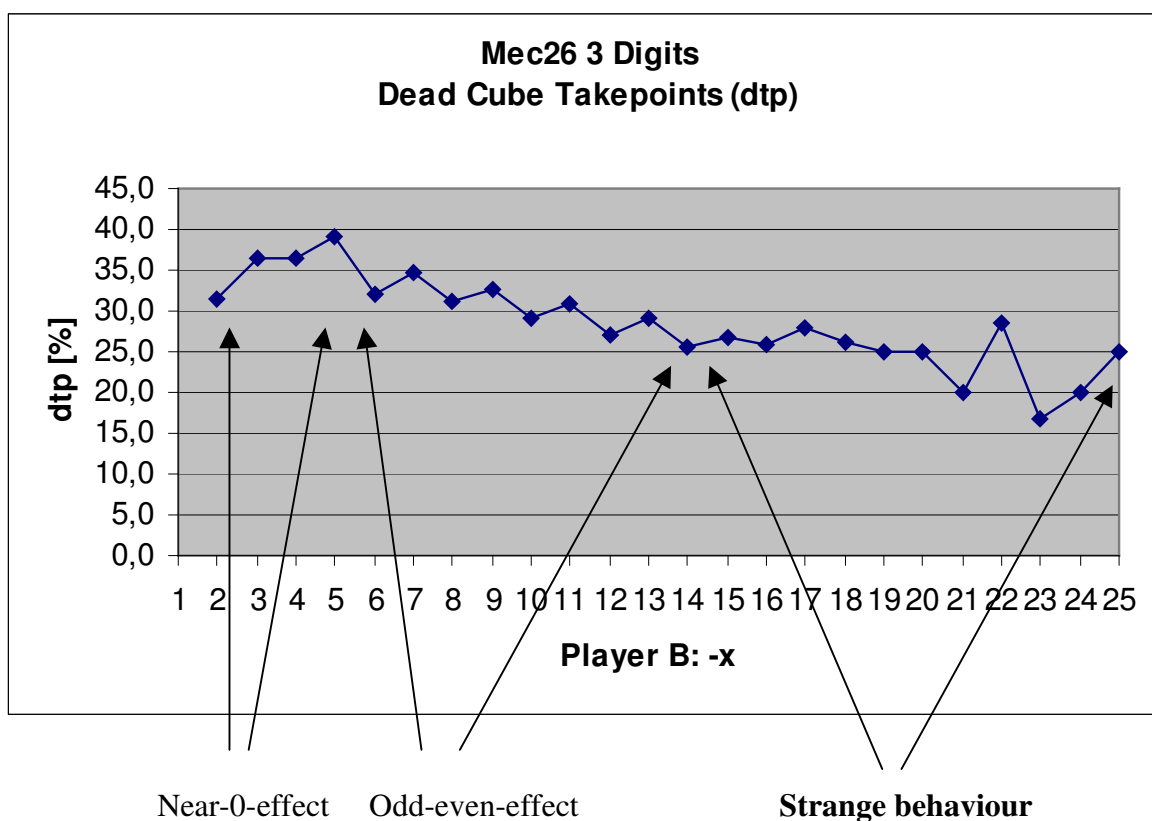


## Rounding errors in match equity tables generating flaws in takepoints

Hello,

some days ago i wrote a script to produce dead and live cube takepoint tables depending on several match equity tables. As basis i used the MET files that were included with GNUBG. After having developed the script and having put the resulting data into an Excel sheet i found some strange behaviour. To avoid errors on my side i compared my results with those in GNUBG theory window (Analyse/Market window). They were identical. Here is an example and some explanation.

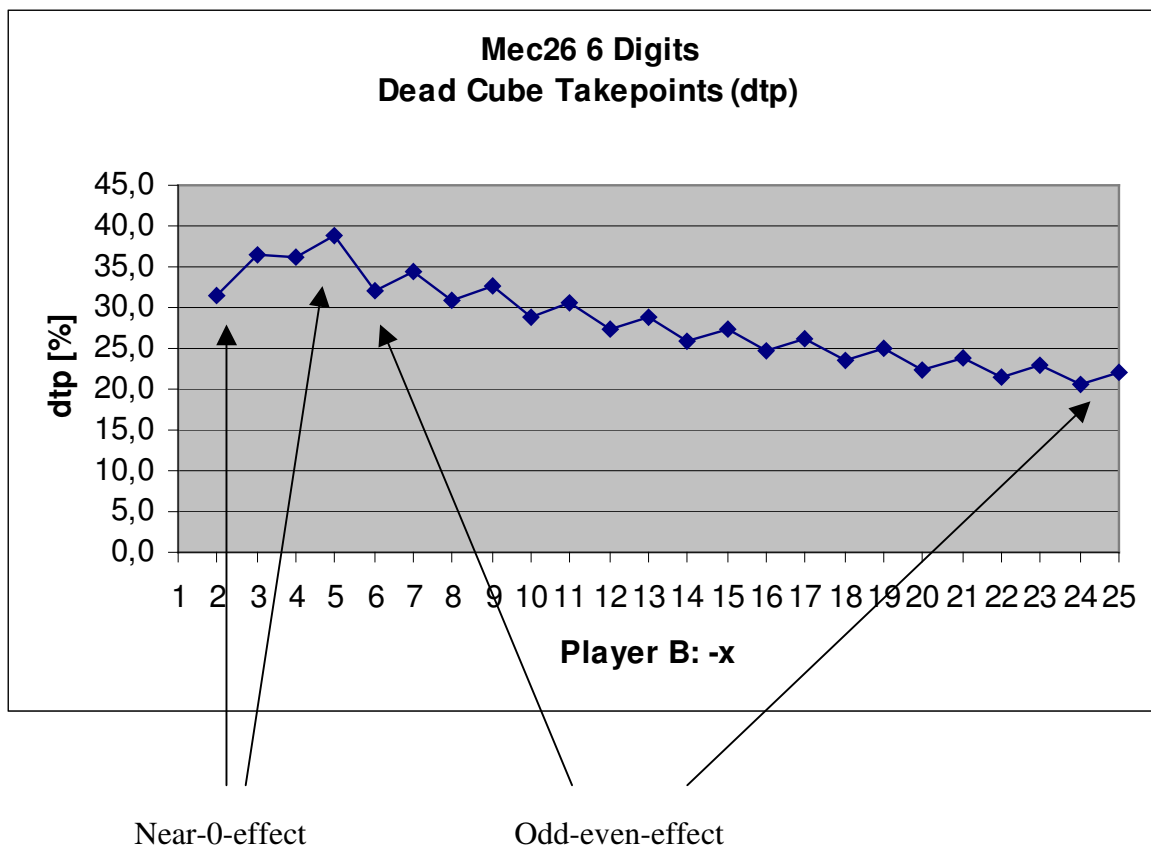
At first I used a Mec26 table with accuracy of 3 digits (see appendix A). These are the numbers produced by the original mec.c. You can see my results for dead cube takepoints as appendix B. Here is the first row of these takepoints as diagram (player A: -2, player B: -x).



What do we expect? With player B being near 0 we don't see any obvious pattern in the diagram. This is what i call the near-0 effect. But when he gets more away from 0 we expect some oscillation resulting from an odd-even effect. With player B being more behind this odd-even effect shouldn't disappear as it does here.

I had the suspicion that the reason was the accuracy of the MET table i was using. I got the mec.c source file. I changed the output format to 6 digits. Now i had a Mec26 with drastically increased accuracy (appendix C). Let's see how the diagram of the takepoints (appendix D) changed.

## Rounding errors in match equity tables generating flaws in takepoints



Now it looks like i expected it. Left side is dominated by near-0-effect, the right side by odd-even. You get the same results if you take the data out of Snowie´s theory window.

**Explanation:** Most backgammon players know how to calculate dead cube takepoints.

**Takepoint = risk/(risk+gain)**

with risk = met(pass) - met(loss)

and gain = met(win) - met(pass)

You may rewrite this as

**Takepoint = (met(pass) - met(loss))/(met(win) - met(loss))**

Now we have a denominator with a difference in it. Dividing by 0 or a number near 0 causes problems. Let´s do a rough estimate how big this error might get.

At first i will estimate the error of the denominator. Mec26 with 3 digits means winning numbers like 25,1 %. 25,149 % is rounded downwards and 25,050 % is rounded upwards. Thus the error per single value is  $\pm 0,050$  %. In the denominator we subtract two numbers and get an overall error of 0,070 % due to Gaussian error propagation  $\Delta(a-b) = \text{sqr}((\Delta a)^2 + (\Delta b)^2)$ .

Now let´s look at the error of the takepoint. Gauss says  $\Delta(a/b) = \Delta a/a + \Delta b/b$  for divisions. This means you have to add the relative errors of numerator and denominator. To simplify matters we will neglect the relative error of the numerator. If the denominator is in the 1 %

## Rounding errors in match equity tables generating flaws in takepoints

range we get a relative error of  $\Delta b/b = 0,070\%/1\% = 7\%$ . The takepoint usually is in the 20% range. Thus a relative error of 7% means  $\pm 1,4\%$  **absolute error**.

This is only a rough estimate. The absolute error even may get bigger if the denominator gets smaller than 1%. Another problem is that there is a non-negligible error for bigger denominators which affects GNUBG's cube handling in a negative way. Live cube takepoints are affected as well.

But there is an easy way out. Just use a more accurate Mec26 table. Even if Mec26 wasn't a perfect match equity table compared to others it is consistent in itself due to its process of creation. The algorithm seems to be trustable. If you use an output of mec.c with 6 digits as i did the estimated absolute error is reduced to 0,002%.

### Conclusion

- Small rounding errors in match equity tables may cause rather big errors when calculating takepoints. This means you have to use match equity tables with sufficient accuracy to get satisfying results. Mec.c gives good results if accuracy is extended to 6 digits.
- Human-made match equity tables like Woolsey's, Zadeh's etc. aren't of adequate accuracy for use with computer programs because their rounding errors are even bigger. The important Woolsey MET may be replaced by a Mec20 table with 6 digits accuracy. It looks pretty the same in most areas but avoids the described problems.

### Future improvements

Computers should use computer-made match equity tables. They are created by algorithms with appropriate input. The most important input is the gammon rate.

Still unresolved is the question what gammon rate is suitable for analyzing human players. Is it 20% like described by Kit Woolsey or is it 26% as the bots do? Maybe some online gaming sites like Gamesgrid or Truemoneygames could provide statistical data.

Another question is how gammon rates behave in „gammon-go“ vs. „gammon-save“ situations? Is the outcome still symmetrical? Extended rollouts may answer this question.

More accurate „fish vs shark“ match equity tables similar to those created by Jake Jacobs would be a further improvement.

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# Rounding errors in match equity tables generating flaws in takepoints

## Appendix C: Mec26 with 6 digits accuracy

1	50.0000	68.5000	75.0000	81.8450	84.2900	88.1877	90.9225	93.6393	94.5938	96.2409	96.8196	97.7822	98.1205	98.6908	98.8911	99.2273	99.3454	99.5439	99.8136	99.7308	99.7719	99.8411	99.8654	99.9062	99.9205
2	31.5000	50.0000	59.4518	66.4121	73.6770	79.4627	83.5040	86.9048	89.5611	91.8765	93.5001	94.9299	95.9480	96.8560	97.4890	98.0549	98.4467	98.7997	99.0422	99.2608	99.4106	99.5457	99.6381	99.7213	99.7782
3	25.0000	40.5482	50.0000	57.1356	64.5982	70.9921	75.7988	80.0177	83.5346	86.6828	89.0246	91.1383	92.7226	94.1705	95.2220	96.1864	96.8855	97.5267	97.9849	98.4044	98.7033	98.9763	99.1699	99.3462	99.4710
4	18.1550	33.5879	42.8644	50.0000	57.4731	64.0480	69.3653	73.9139	78.0668	81.7134	84.6814	87.2486	89.3710	91.2255	92.7132	94.0079	95.0486	95.9540	96.6689	97.2876	97.7747	98.1952	98.5238	98.8064	99.0266
5	15.7500	26.3200	35.4018	42.5269	50.0000	56.6520	62.3245	67.3423	71.9650	76.1077	79.5917	82.6909	85.2993	87.6321	89.5441	91.2458	92.6432	93.8809	94.8802	95.7601	96.4677	97.0879	97.5830	98.0145	98.3680
6	10.8124	20.5373	29.0079	35.9520	43.3480	50.0000	55.9452	61.1751	66.1514	70.6256	74.5622	78.0448	81.1029	83.8197	86.1465	88.1994	89.9574	91.5038	92.8055	93.9427	94.8944	95.7223	96.4094	97.0039	97.4949
7	9.0775	16.4960	24.2012	30.6347	37.6755	44.0548	50.0000	55.3238	60.4782	65.1647	69.4050	73.2138	76.6274	79.6989	82.3669	84.7886	86.8866	88.7527	90.3659	91.7719	92.9799	94.0416	94.9384	95.7237	96.3834
8	6.3607	13.0962	19.9823	26.0861	32.6577	38.8249	44.6762	50.0000	55.2022	60.0129	64.4592	68.4951	72.1664	75.5351	78.5330	81.2281	83.6332	85.7871	87.6789	89.3691	90.8236	92.1174	93.2349	94.2168	95.0679
9	5.4052	10.4389	16.4654	21.9332	28.0360	33.8486	39.5218	44.7978	50.0000	54.8772	59.4562	63.6815	67.5928	71.1900	74.4536	77.4264	80.1128	82.5459	84.7129	86.6592	88.3787	89.9136	91.2573	92.4487	93.4651
10	3.7591	8.1235	13.3172	18.2866	23.8923	29.3745	34.8353	39.9871	45.1228	50.0000	54.6541	58.9928	63.0699	66.8519	70.3380	73.5363	76.4713	79.1496	81.5710	83.7625	85.7268	87.4934	89.0624	90.4636	91.6986
11	3.1804	6.9999	10.9754	15.3186	20.4083	25.4378	30.5960	35.5408	40.5408	45.3459	50.0000	54.3931	58.5731	62.4926	66.1502	69.5436	72.6880	75.5866	78.2377	80.6601	82.8568	84.8486	86.6383	88.2510	89.6890
12	2.2178	5.0701	8.8617	12.7514	17.3091	21.9652	26.7962	31.5049	36.3185	41.0072	45.6069	50.0000	54.2280	58.2318	62.0132	65.5527	68.8676	71.9500	74.8010	77.4278	79.8349	82.0371	84.0366	85.8523	87.4887
13	1.8795	4.0520	7.2774	10.6290	14.7007	18.8971	23.3726	27.8136	32.4072	36.9301	41.4269	45.7720	50.0000	54.0445	57.9048	61.5535	65.0031	68.2367	71.2605	74.0689	76.6669	79.0634	81.2600	83.2709	85.0999
14	1.3092	3.1420	5.8295	8.7745	12.3679	16.1803	20.3011	24.4649	28.8100	33.1481	37.5074	41.7682	45.9555	50.0000	53.8993	57.6174	61.1639	64.5183	67.6797	70.6416	73.4061	75.9760	78.3526	80.5447	82.5561
15	1.1089	2.5110	4.7780	7.2668	10.4559	13.8535	17.6131	21.4670	25.5464	29.6620	33.8496	37.9668	42.0952	46.1007	50.0000	53.7513	57.3604	60.8012	64.0720	67.1804	70.0670	72.7895	75.3279	77.6868	79.8685
16	0.7727	1.9451	3.8136	5.9921	8.7542	11.8006	15.2114	18.7179	22.5736	26.4617	30.4564	34.4473	38.4465	42.3826	46.2487	50.0000	53.6379	57.1338	60.4834	63.6702	66.6928	69.5445	72.2240	74.7316	77.0686
17	0.6546	1.5533	3.1145	4.9514	7.3568	10.0426	13.1134	16.3668	19.8872	23.5287	27.3120	31.1324	34.9969	38.8361	42.6396	46.3621	50.0000	53.5228	56.9234	60.1829	63.2968	66.2566	69.0557	71.6943	74.1707
18	0.4561	1.2003	2.4733	4.0460	6.1191	8.4962	11.2473	14.2129	17.4541	20.8504	24.4134	28.0500	31.7613	35.4817	39.1988	42.8662	46.4772	50.0000	53.4253	56.7315	59.9123	62.9649	65.8542	68.6042	71.2026
19	0.3864	0.9578	2.0151	3.3311	5.1198	7.1945	9.6441	12.3211	15.2871	18.4290	21.7623	25.1990	28.7395	32.3203	35.9280	39.5186	43.0766	46.5747	50.0000	53.3291	56.5533	59.6577	62.6353	65.4775	68.1803
20	0.2891	0.7392	1.5956	2.7124	4.2399	6.0573	8.2281	10.6409	13.3408	16.2375	19.3399	22.5722	25.9311	29.3864	32.8396	36.3298	39.8171	43.2895	46.6709	50.0000	53.2451	56.3896	59.4245	62.3393	65.1281
21	0.2281	0.5694	1.2967	2.2523	3.5323	5.1056	7.0201	9.1764	11.6213	14.2732	17.1442	20.1651	23.3331	26.5939	29.9330	33.3072	36.7032	40.0877	43.4467	46.7549	50.0000	53.1639	56.2363	59.2046	62.0614
22	0.1589	0.4543	1.0237	1.8048	2.9121	4.2777	5.9584	7.8826	10.0864	12.5066	15.1514	17.9629	20.9366	24.0240	27.2105	30.4555	33.7444	37.0451	40.3423	43.6104	46.8361	50.0000	53.0805	56.0937	59.0005
23	0.11346	0.3619	0.8301	1.4762	2.4170	3.5906	5.0606	6.7851	8.7427	10.9376	13.3617	15.9634	18.7400	21.6474	24.6721	27.7760	30.9443	34.1498	37.3647	40.5795	43.7637	46.9095	50.0000	53.0201	55.9596
24	0.0938	0.2787	0.6538	1.1936	1.9955	2.9961	4.2763	5.7842	7.5364	11.7490	14.1477	16.7291	19.4553	22.3132	25.2684	28.3057	31.3958	34.5225	37.6607	40.7954	43.9063	46.9799	50.0000	52.9654	
25	0.0795	0.2218	0.5290	0.9734	1.6420	2.5051	3.6166	4.9421	6.5149	8.3004	10.3110	12.5113	14.9001	17.4439	20.1315	22.9314	25.8293	28.7974	31.8197	34.8719	37.9386	40.9995	44.0494	47.0446	50.0000

# Rounding errors in match equity tables generating flaws in takepoints

## Appendix D: Dead Cube Takepoints derived from Mec26 with 6 digits accuracy

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
2	x	31.5	36.5	36.3	38.8	32.2	34.5	31.0	32.8	28.7	30.5	27.3	28.9	25.8	27.4	24.6	26.1	23.4	24.9	22.4	23.8	21.5	22.8	20.6	22.0
3	x	27.0	31.1	37.4	30.9	30.3	28.2	29.7	26.8	26.9	25.0	25.7	23.9	24.3	22.7	23.2	21.8	22.2	20.9	21.3	20.2	20.6	19.6	19.9	19.0
4	x	18.9	22.9	28.3	29.5	28.8	29.6	30.1	29.8	28.9	29.0	28.7	28.6	28.0	27.9	27.4	27.2	26.6	26.5	25.9	25.8	25.2	25.1	24.6	24.5
5	x	17.2	20.8	23.2	24.4	24.4	24.9	26.1	25.8	25.7	25.5	25.9	25.5	25.7	25.3	25.5	25.0	25.0	24.6	24.7	24.3	24.3	23.9	23.9	23.5
6	x	21.6	23.3	25.4	26.3	26.3	27.1	27.5	27.6	27.3	27.5	27.4	27.5	27.3	27.3	27.1	27.0	26.7	26.7	26.4	26.3	26.0	25.9	25.7	25.5
7	x	22.0	24.3	25.3	25.8	25.6	25.9	26.5	26.5	26.4	26.3	26.5	26.4	26.4	26.3	26.4	26.2	26.1	26.0	25.9	25.7	25.7	25.5	25.4	25.2
8	x	19.7	21.2	23.3	24.3	24.9	25.5	26.2	26.4	26.5	26.6	26.8	26.9	26.9	26.9	26.9	26.9	26.8	26.7	26.6	26.5	26.4	26.3	26.2	26.1
9	x	20.6	22.4	22.5	23.7	23.6	24.2	24.8	25.2	25.3	25.5	25.7	25.9	26.0	26.1	26.2	26.2	26.2	26.2	26.1	26.2	26.1	26.1	26.0	25.9
10	x	20.3	21.7	23.1	23.9	24.2	24.8	25.2	25.5	25.6	25.9	26.0	26.2	26.3	26.4	26.4	26.5	26.5	26.5	26.4	26.4	26.4	26.3	26.3	26.2
11	x	22.2	23.7	23.6	24.3	24.1	24.5	24.8	25.1	25.2	25.4	25.6	25.7	25.8	25.9	26.0	26.1	26.1	26.1	26.2	26.2	26.2	26.1	26.1	26.1
12	x	20.0	21.1	22.5	23.2	23.7	24.2	24.7	25.0	25.2	25.4	25.6	25.8	25.9	26.0	26.1	26.2	26.2	26.2	26.3	26.3	26.3	26.3	26.3	26.2
13	x	22.0	23.2	22.8	23.6	23.5	24.0	24.2	24.6	24.7	25.0	25.2	25.3	25.5	25.6	25.7	25.8	25.9	26.0	26.0	26.1	26.1	26.1	26.1	26.1
14	x	20.1	21.0	22.1	22.7	23.2	23.7	24.1	24.4	24.6	24.9	25.1	25.3	25.4	25.6	25.7	25.8	25.9	26.0	26.0	26.1	26.1	26.1	26.1	26.1
15	x	22.5	23.5	22.8	23.5	23.4	23.8	24.0	24.3	24.5	24.7	24.9	25.0	25.2	25.3	25.5	25.6	25.7	25.8	25.8	25.9	25.9	26.0	26.0	26.0
16	x	20.1	20.8	21.8	22.4	22.8	23.3	23.7	24.0	24.2	24.5	24.7	24.9	25.1	25.2	25.4	25.5	25.6	25.7	25.8	25.8	25.9	25.9	26.0	26.0
17	x	22.5	23.4	22.6	23.2	23.1	23.4	23.6	23.9	24.1	24.3	24.5	24.7	24.8	25.0	25.2	25.3	25.4	25.5	25.6	25.7	25.7	25.8	25.9	25.9
18	x	20.1	20.8	21.7	22.2	22.7	23.1	23.4	23.7	23.9	24.2	24.4	24.6	24.7	24.9	25.1	25.2	25.3	25.4	25.5	25.6	25.7	25.7	25.8	25.8
19	x	22.7	23.5	22.7	23.2	23.0	23.3	23.5	23.8	23.9	24.1	24.3	24.4	24.6	24.8	24.9	25.0	25.2	25.3	25.4	25.5	25.5	25.6	25.7	25.7
20	x	20.2	20.8	21.6	22.1	22.5	22.9	23.2	23.5	23.7	23.9	24.1	24.3	24.5	24.7	24.8	24.9	25.1	25.2	25.3	25.4	25.5	25.5	25.6	25.7
21	x	22.8	23.5	22.6	23.0	22.9	23.2	23.3	23.5	23.7	23.9	24.0	24.2	24.4	24.5	24.7	24.8	24.9	25.0	25.1	25.2	25.3	25.4	25.5	25.6
22	x	20.3	20.8	21.6	22.0	22.4	22.7	23.0	23.3	23.5	23.7	23.9	24.1	24.3	24.4	24.6	24.7	24.8	24.9	25.0	25.2	25.2	25.3	25.4	25.5
23	x	22.9	23.5	22.6	23.0	22.8	23.1	23.2	23.4	23.5	23.7	23.9	24.0	24.2	24.3	24.5	24.6	24.7	24.8	24.9	25.0	25.1	25.2	25.3	25.4
24	x	20.3	20.8	21.5	21.9	22.3	22.6	22.9	23.1	23.3	23.5	23.7	23.9	24.1	24.2	24.4	24.5	24.6	24.7	24.8	25.0	25.0	25.1	25.2	25.3
25	x	23.0	23.5	22.5	22.9	22.7	22.9	23.0	23.2	23.4	23.6	23.7	23.9	24.0	24.1	24.3	24.4	24.5	24.6	24.7	24.9	24.9	25.0	25.1	25.2