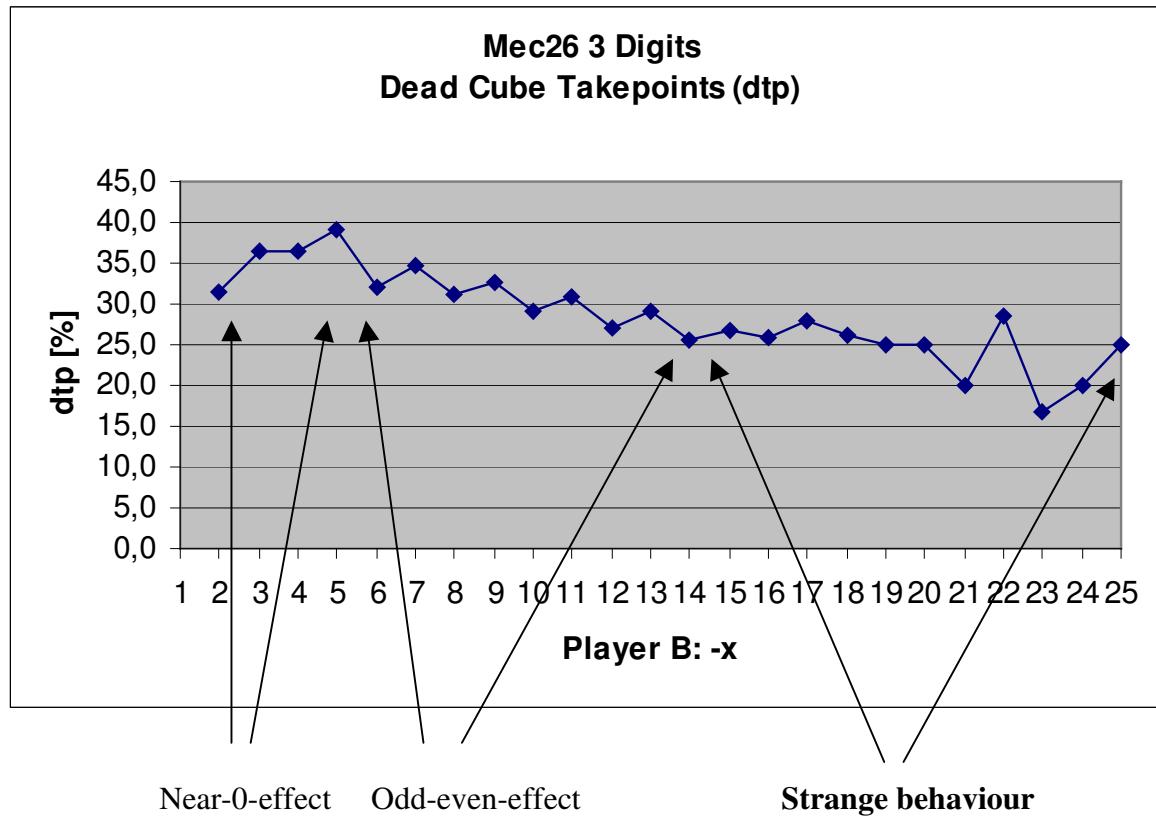


Rounding errors in match equity tables generating flaws in takepoints

Hello,

some days ago i wrote a script to produce dead and live cube takepoint tables depending on several match equity tables. As basis i used the MET files that were included with GNUMBG. After having developed the script and having put the resulting data into an Excel sheet i found some strange behaviour. To avoid errors on my side i compared my results with those in GNUMBG theory window (Analyse/Market window). They were identical. Here is an example and some explanation.

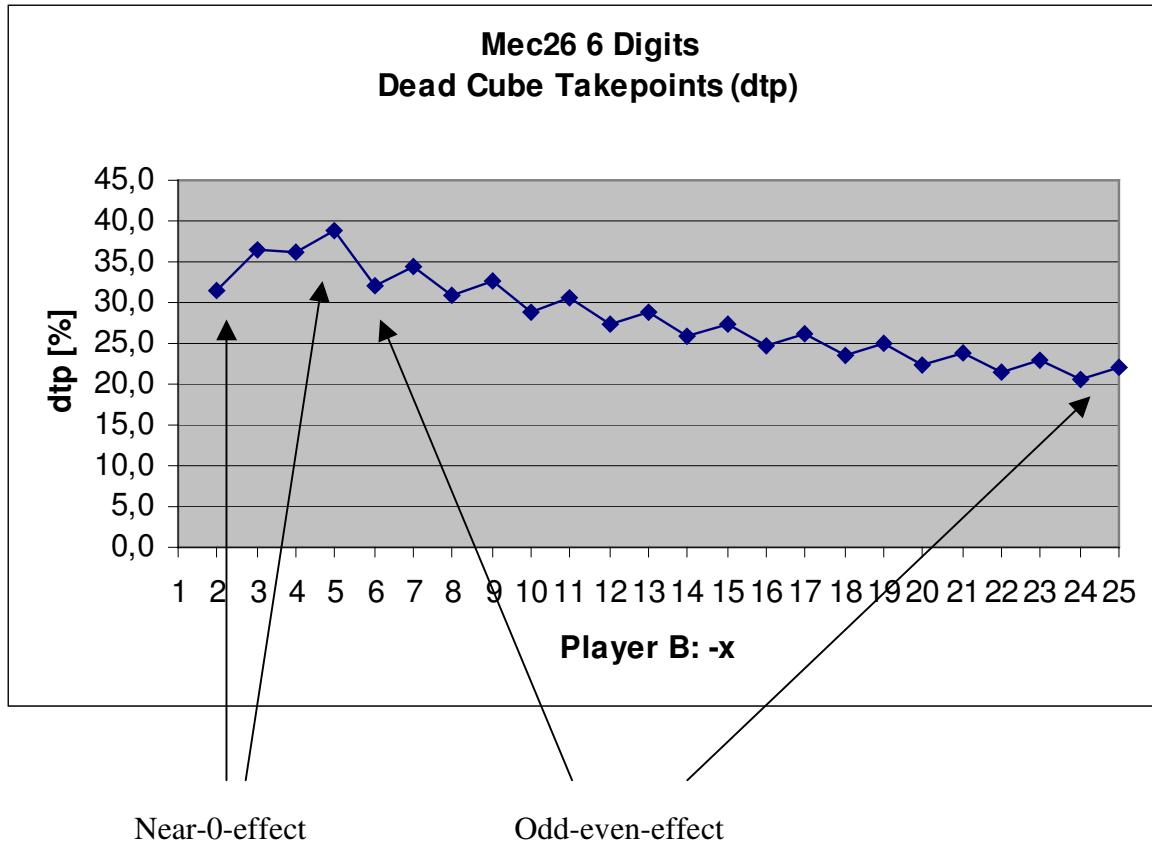
At first I used a Mec26 table with accuracy of 3 digits (see appendix A). These are the numbers produced by the original `mec.c`. You can see my results for dead cube takepoints as appendix B. Here is the first row of these takepoints as diagram (player A: -2, player B: -x).



What do we expect? With player B being near 0 we don't see any obvious pattern in the diagram. This is what i call the near-0 effect. But when he gets more away from 0 we expect some oscillation resulting from an odd-even effect. With player B being more behind this odd-even effect shouldn't disappear as it does here.

I had the suspicion that the reason was the accuracy of the MET table i was using. I got the `mec.c` source file. I changed the output format to 6 digits. Now i had a Mec26 with drastically increased accuracy (appendix C). Let's see how the diagram of the takepoints (appendix D) changed.

Rounding errors in match equity tables generating flaws in takepoints



Now it looks like i expected it. Left side is dominated by near-0-effect, the right side by odd-even. You get the same results if you take the data out of Snowie's theory window.

Explanation: Most backgammon players know how to calculate dead cube takepoints.

Takepoint = risk/(risk+gain)

with risk = met(pass) - met(loss)
and gain = met(win) - met(pass)

You may rewrite this as

Takepoint = (met(pass) - met(loss))/(met(win) - met(loss))

Now we have a denominator with a difference in it. Dividing by 0 or a number near 0 causes problems. Let's do a rough estimate how big this error might get.

At first i will estimate the error of the denominator. Mec26 with 3 digits means winning numbers like 25,1 %. 25,149 % is rounded downwards and 25,050 % is rounded upwards. Thus the error per single value is $\pm 0,050 \%$. In the denominator we subtract two numbers and get an overall error of 0,070 % due to Gaussian error propagation $\Delta(a-b) = \text{sqr}((\Delta a)^2 + (\Delta b)^2)$.

Now let's look at the error of the takepoint. Gauss says $\Delta(a/b) = \Delta a/a + \Delta b/b$ for divisions. This means you have to add the relative errors of enumerator and denominator. To simplify matters we will neglect the relative error of the enumerator. If the denominator is in the 1 %

Rounding errors in match equity tables generating flaws in takepoints

range we get a relative error of $\Delta b/b = 0,070\%/1\% = 7\%$. The takepoint usually is in the 20 % range. Thus a relative error of 7 % means **± 1,4 % absolute error**.

This is only a rough estimate. The absolute error even may get bigger if the denominator gets smaller than 1 %. Another problem is that there is a non-negligible error for bigger denominators which affects GNUBG's cube handling in a negative way. Live cube takepoints are affected as well.

But there is an easy way out. Just use a more accurate Mec26 table. Even if Mec26 wasn't a perfect match equity table compared to others it is consistent in itself due to its process of creation. The algorithm seems to be trustable. If you use an output of `mec.c` with 6 digits as input the estimated absolute error is reduced to 0,002 %.

Conclusion

- Small rounding errors in match equity tables may cause rather big errors when calculating takepoints. This means you have to use match equity tables with sufficient accuracy to get satisfying results. `Mec.c` gives good results if accuracy is extended to 6 digits.
- Human-made match equity tables like Woolsey's, Zadeh's etc. aren't of adequate accuracy for use with computer programs because their rounding errors are even bigger. The important Woolsey MET may be replaced by a Mec20 table with 6 digits accuracy. It looks pretty the same in most areas but avoids the described problems.

Future improvements

Computers should use computer-made match equity tables. They are created by algorithms with appropriate input. The most important input is the gammon rate.

Still unresolved is the question what gammon rate is suitable for analyzing human players. Is it 20 % like described by Kit Woolsey or is it 26 % as the bots do? Maybe some online gaming sites like Gamesgrid or Truemoneygames could provide statistical data.

Another question is how gammon rates behave in „gammon-go“ vs. „gammon-save“ situations? Is the outcome still symmetrical? Extended rollouts may answer this question.

More accurate „fish vs shark“ match equity tables similar to those created by Jake Jacobs would be a further improvement.

Joachim Matussek
aka weaky on FIBS

Graduate physicist
joachim.matussek@web.de

Rounding errors in match equity tables generating flaws in takepoints

Appendix A: Mec26 with 3 digits accuracy

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | 50,0 | 68,5 | 75,0 | 81,8 | 84,3 | 89,2 | 90,9 | 93,6 | 94,6 | 96,2 | 96,8 | 97,8 | 98,1 | 98,7 | 98,9 | 99,2 | 99,3 | 99,5 | 99,6 | 99,7 | 99,8 | 99,8 | 99,9 | 99,9 | 99,9 |
| 2 | 31,5 | 50,0 | 59,5 | 66,4 | 73,7 | 79,5 | 83,5 | 86,9 | 89,6 | 91,9 | 93,5 | 94,9 | 95,9 | 96,9 | 97,5 | 98,1 | 98,4 | 98,8 | 99,0 | 99,3 | 99,4 | 99,5 | 99,6 | 99,7 | 99,8 |
| 3 | 25,0 | 40,5 | 50,0 | 57,1 | 64,6 | 71,0 | 75,8 | 80,0 | 83,5 | 86,7 | 89,0 | 91,1 | 92,7 | 94,2 | 95,2 | 96,2 | 96,9 | 97,5 | 98,0 | 98,4 | 98,7 | 99,0 | 99,2 | 99,3 | 99,5 |
| 4 | 18,2 | 33,6 | 42,9 | 50,0 | 57,5 | 64,0 | 69,4 | 73,9 | 78,1 | 81,7 | 84,7 | 87,2 | 89,4 | 91,2 | 92,7 | 94,0 | 95,0 | 96,0 | 96,7 | 97,3 | 97,8 | 98,2 | 98,5 | 98,8 | 99,0 |
| 5 | 15,8 | 26,3 | 35,4 | 42,5 | 50,0 | 56,7 | 62,3 | 67,3 | 72,0 | 76,1 | 79,6 | 82,7 | 85,3 | 87,6 | 89,5 | 91,2 | 92,6 | 93,9 | 94,9 | 95,8 | 96,5 | 97,1 | 97,6 | 98,0 | 98,4 |
| 6 | 10,8 | 20,5 | 29,0 | 36,0 | 43,3 | 50,0 | 55,9 | 61,2 | 66,2 | 70,6 | 74,6 | 78,0 | 81,1 | 83,8 | 86,1 | 88,2 | 90,0 | 91,5 | 92,8 | 93,9 | 94,9 | 95,7 | 96,4 | 97,0 | 97,5 |
| 7 | 9,1 | 16,5 | 24,2 | 30,6 | 37,7 | 44,1 | 50,0 | 55,3 | 60,5 | 65,2 | 69,4 | 73,2 | 76,6 | 79,7 | 82,4 | 84,8 | 86,9 | 88,8 | 90,4 | 91,8 | 93,0 | 94,0 | 94,9 | 95,7 | 96,4 |
| 8 | 6,4 | 13,1 | 20,0 | 26,1 | 32,7 | 38,8 | 44,7 | 50,0 | 55,2 | 60,0 | 64,5 | 68,5 | 72,2 | 75,5 | 78,5 | 81,2 | 83,6 | 85,8 | 87,7 | 89,4 | 90,8 | 92,1 | 93,2 | 94,2 | 95,1 |
| 9 | 5,4 | 10,4 | 16,5 | 21,9 | 28,0 | 33,8 | 39,5 | 44,8 | 50,0 | 54,9 | 59,5 | 63,7 | 67,6 | 71,2 | 74,5 | 77,4 | 80,1 | 82,5 | 84,7 | 86,7 | 88,4 | 89,9 | 91,3 | 92,4 | 93,5 |
| 10 | 3,8 | 8,1 | 13,3 | 18,3 | 23,9 | 29,4 | 34,8 | 40,0 | 45,1 | 50,0 | 54,7 | 59,0 | 63,1 | 66,9 | 70,3 | 73,5 | 76,5 | 79,1 | 81,6 | 83,8 | 85,7 | 87,5 | 89,1 | 90,5 | 91,7 |
| 11 | 3,2 | 6,5 | 11,0 | 15,3 | 20,4 | 25,4 | 30,6 | 35,5 | 40,5 | 45,3 | 50,0 | 54,4 | 58,6 | 62,5 | 66,2 | 69,5 | 72,7 | 75,6 | 78,2 | 80,7 | 82,9 | 84,8 | 86,6 | 88,3 | 89,7 |
| 12 | 2,2 | 5,1 | 8,9 | 12,8 | 17,3 | 22,0 | 26,8 | 31,5 | 36,3 | 41,0 | 45,6 | 50,0 | 54,2 | 58,2 | 62,0 | 65,6 | 68,9 | 72,0 | 74,8 | 77,4 | 79,8 | 82,0 | 84,0 | 85,9 | 87,5 |
| 13 | 1,9 | 4,1 | 7,3 | 10,6 | 14,7 | 18,9 | 23,4 | 27,8 | 32,4 | 36,9 | 41,4 | 45,8 | 50,0 | 54,0 | 57,9 | 61,6 | 65,0 | 68,2 | 71,3 | 74,1 | 76,7 | 79,1 | 81,3 | 83,3 | 85,1 |
| 14 | 1,3 | 3,1 | 5,8 | 8,8 | 12,4 | 16,2 | 20,3 | 24,5 | 28,8 | 33,1 | 37,5 | 41,8 | 46,0 | 50,0 | 53,9 | 57,6 | 61,2 | 64,5 | 67,7 | 70,6 | 73,4 | 76,0 | 78,4 | 80,5 | 82,6 |
| 15 | 1,1 | 2,5 | 4,8 | 7,3 | 10,5 | 13,9 | 17,6 | 21,5 | 25,5 | 29,7 | 33,8 | 38,0 | 42,1 | 46,1 | 50,0 | 53,8 | 57,4 | 60,8 | 64,1 | 67,2 | 70,1 | 72,8 | 75,3 | 77,7 | 79,9 |
| 16 | 0,8 | 1,9 | 3,8 | 6,0 | 8,8 | 11,8 | 15,2 | 18,8 | 22,6 | 26,5 | 30,5 | 34,4 | 38,4 | 42,4 | 46,2 | 50,0 | 53,6 | 57,1 | 60,5 | 63,7 | 66,7 | 69,5 | 72,2 | 74,7 | 77,1 |
| 17 | 0,7 | 1,6 | 3,1 | 5,0 | 7,4 | 10,0 | 13,1 | 16,4 | 19,9 | 23,5 | 27,3 | 31,1 | 35,0 | 38,8 | 42,6 | 46,4 | 50,0 | 53,5 | 56,9 | 60,2 | 63,3 | 66,3 | 69,1 | 71,7 | 74,2 |
| 18 | 0,5 | 1,2 | 2,5 | 4,0 | 6,1 | 8,5 | 11,2 | 14,2 | 17,5 | 20,9 | 24,4 | 28,0 | 31,8 | 35,5 | 39,2 | 42,9 | 46,5 | 50,0 | 53,4 | 56,7 | 59,9 | 63,0 | 65,9 | 68,6 | 71,2 |
| 19 | 0,4 | 1,0 | 2,0 | 3,3 | 5,1 | 7,2 | 9,6 | 12,3 | 15,3 | 18,4 | 21,8 | 25,2 | 28,7 | 32,3 | 35,9 | 39,5 | 43,1 | 46,6 | 50,0 | 53,3 | 56,6 | 59,7 | 62,6 | 65,5 | 68,2 |
| 20 | 0,3 | 0,7 | 1,6 | 2,7 | 4,2 | 6,1 | 8,2 | 10,6 | 13,3 | 16,2 | 19,3 | 22,6 | 25,9 | 29,4 | 32,8 | 36,3 | 39,8 | 43,3 | 46,7 | 50,0 | 53,2 | 56,4 | 59,4 | 62,3 | 65,1 |
| 21 | 0,2 | 0,6 | 1,3 | 2,2 | 3,5 | 5,1 | 7,0 | 9,2 | 11,6 | 14,3 | 17,1 | 20,2 | 23,3 | 26,6 | 29,9 | 33,3 | 36,7 | 40,1 | 43,4 | 46,8 | 50,0 | 53,2 | 56,2 | 59,2 | 62,1 |
| 22 | 0,2 | 0,5 | 1,0 | 1,8 | 2,9 | 4,3 | 6,0 | 7,9 | 10,1 | 12,5 | 15,2 | 18,0 | 20,9 | 24,0 | 27,2 | 30,5 | 33,7 | 37,0 | 40,3 | 43,6 | 46,8 | 50,0 | 53,1 | 56,1 | 59,0 |
| 23 | 0,1 | 0,4 | 0,8 | 1,5 | 2,4 | 3,6 | 5,1 | 6,8 | 8,7 | 10,9 | 13,4 | 16,0 | 18,7 | 21,6 | 24,7 | 27,8 | 30,9 | 34,1 | 37,4 | 40,6 | 43,8 | 46,9 | 50,0 | 53,0 | 56,0 |
| 24 | 0,1 | 0,3 | 0,7 | 1,2 | 2,0 | 3,0 | 4,3 | 5,8 | 7,6 | 9,5 | 11,7 | 14,1 | 16,7 | 19,5 | 22,3 | 25,3 | 28,3 | 31,4 | 34,5 | 37,7 | 40,8 | 43,9 | 47,0 | 50,0 | 53,0 |
| 25 | 0,1 | 0,2 | 0,5 | 1,0 | 1,6 | 2,5 | 3,6 | 4,9 | 6,5 | 8,3 | 10,3 | 12,5 | 14,9 | 17,4 | 20,1 | 22,9 | 25,8 | 28,8 | 31,8 | 34,9 | 37,9 | 41,0 | 44,0 | 47,0 | 50,0 |

Appendix B: Dead Cube Takepoints derived from Mec26 with 3 digits accuracy

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | |
|----|---|------|------|------|------|------|------|------|------|------|------|------|------|------|------|----------|------|------|------|------|------|------|------|------|------|------|
| 1 | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | | |
| 2 | x | 31,5 | 36,5 | 36,4 | 39,0 | 32,1 | 34,6 | 31,2 | 32,7 | 29,0 | 30,8 | 27,2 | 29,2 | 25,5 | 26,8 | 25,8 | 28,0 | 26,3 | 25,0 | 25,0 | 20,0 | 28,6 | 16,7 | 20,0 | 25,0 | |
| 3 | x | 27,0 | 31,0 | 37,3 | 30,7 | 30,2 | 28,1 | 29,6 | 26,6 | 26,5 | 24,8 | 26,1 | 24,2 | 23,7 | 22,6 | 22,0 | 22,0 | 21,2 | 22,2 | 18,2 | 22,2 | 21,4 | 25,0 | 22,2 | 14,3 | |
| 4 | x | 19,0 | 23,0 | 28,4 | 29,5 | 28,8 | 29,6 | 30,1 | 30,2 | 28,9 | 29,2 | 28,8 | 28,6 | 27,8 | 28,4 | 27,5 | 26,3 | 27,1 | 25,0 | 27,3 | 25,9 | 22,7 | 22,2 | 26,7 | 23,1 | |
| 5 | x | 17,0 | 20,7 | 23,1 | 24,3 | 24,6 | 24,8 | 26,2 | 25,5 | 25,8 | 25,3 | 26,0 | 25,2 | 26,1 | 25,3 | 25,6 | 25,7 | 23,8 | 24,1 | 24,4 | 23,7 | 25,0 | 25,9 | 22,7 | 26,3 | |
| 6 | x | 21,7 | 23,4 | 25,4 | 26,3 | 26,1 | 27,2 | 27,6 | 27,5 | 27,3 | 27,6 | 27,1 | 27,7 | 27,3 | 27,6 | 27,5 | 27,0 | 26,9 | 26,9 | 25,9 | 26,0 | 25,6 | 25,0 | 25,8 | 23,1 | |
| 7 | x | 22,1 | 24,3 | 25,0 | 26,0 | 25,7 | 26,0 | 26,3 | 26,4 | 26,4 | 26,2 | 26,9 | 26,4 | 26,4 | 26,4 | 26,4 | 25,5 | 26,4 | 26,3 | 26,3 | 27,1 | 26,2 | 26,4 | 26,1 | 25,0 | |
| 8 | x | 19,5 | 21,2 | 23,6 | 24,0 | 24,7 | 25,4 | 26,3 | 26,5 | 26,2 | 26,8 | 26,7 | 27,1 | 26,8 | 26,6 | 26,8 | 27,0 | 26,2 | 26,1 | 25,9 | 26,4 | 27,0 | 26,8 | 26,5 | 25,6 | |
| 9 | x | 20,6 | 22,3 | 22,3 | 23,6 | 23,9 | 24,1 | 24,7 | 24,7 | 25,2 | 25,5 | 25,3 | 25,7 | 25,7 | 26,3 | 26,4 | 26,5 | 26,6 | 26,3 | 26,2 | 25,8 | 26,5 | 26,0 | 26,2 | 25,5 | |
| 10 | x | 20,6 | 21,6 | 23,3 | 24,2 | 24,4 | 25,0 | 25,2 | 25,5 | 25,5 | 25,8 | 25,9 | 26,3 | 26,1 | 26,0 | 26,6 | 26,3 | 26,8 | 26,8 | 26,2 | 26,1 | 26,5 | 26,1 | 26,2 | 26,7 | |
| 11 | x | 22,1 | 24,1 | 23,4 | 24,1 | 23,8 | 24,6 | 24,7 | 25,3 | 25,3 | 25,3 | 25,5 | 25,6 | 25,6 | 26,4 | 26,2 | 25,9 | 26,2 | 25,8 | 26,1 | 26,5 | 26,1 | 26,2 | 25,0 | 26,1 | |
| 12 | x | 19,8 | 20,7 | 22,7 | 22,3 | 24,1 | 24,0 | 25,0 | 25,1 | 25,4 | 25,5 | 25,6 | 25,7 | 26,0 | 25,5 | 26,1 | 26,2 | 25,9 | 26,8 | 26,3 | 25,7 | 26,7 | 26,9 | 25,9 | 26,0 | 26,0 |
| 13 | x | 21,5 | 23,1 | 22,3 | 23,7 | 23,0 | 23,9 | 24,1 | 24,6 | 24,6 | 25,0 | 25,1 | 25,6 | 25,7 | 25,9 | 25,2 | 25,7 | 25,7 | 25,8 | 26,4 | 26,7 | 26,2 | 26,3 | 26,1 | 26,2 | 26,2 |
| 14 | x | 19,6 | 21,1 | 22,7 | 22,6 | 23,5 | 23,6 | 24,2 | 24,4 | 24,8 | 25,0 | 24,9 | 25,1 | 25,6 | 25,6 | 25,6 | 26,0 | 26,4 | 25,7 | 25,6 | 25,6 | 25,4 | 25,5 | 26,3 | 26,4 | 26,4 |
| 15 | x | 24,4 | 24,2 | 22,2 | 23,2 | 23,3 | 24,0 | 23,7 | 24,3 | 24,7 | 24,5 | 24,8 | 24,7 | 25,0 | 25,3 | 25,8 | 25,3 | 25,7 | 25,9 | 26,3 | 26,2 | 26,1 | 25,9 | 26,7 | 25,5 | 25,5 |
| 16 | x | 19,4 | 20,0 | 21,7 | 22,1 | 22,5 | 23,5 | 23,6 | 24,3 | 23,8 | 24,8 | 24,8 | 25,2 | 25,0 | 25,2 | 25,0 | 25,3 | 25,5 | 25,5 | 25,2 | 25,6 | 26,0 | 26,5 | 25,5 | 26,0 | 26,0 |
| 17 | x | 24,0 | 24,4 | 22,8 | 23,0 | 23,6 | 23,5 | 23,5 | 23,4 | 24,1 | 23,7 | 24,8 | 25,0 | 24,7 | 25,3 | 25,3 | 25,7 | 25,7 | 25,5 | 25,5 | 25,5 | 25,8 | 26,2 | 25,8 | 26,3 | 25,9 |
| 18 | x | 15,8 | 21,2 | 20,8 | 22,2 | 23,1 | 23,1 | 23,3 | 23,7 | 24,4 | 24,6 | 24,6 | 24,4 | 24,3 | 25,0 | 24,8 | 25,0 | 24,8 | 25,4 | 25,7 | 25,5 | 25,6 | 25,0 | 25,2 | 25,6 | 25,9 |
| 19 | x | 25,0 | 22,2 | 25,0 | 24,1 | 22,4 | 23,7 | 23,9 | 23,3 | 23,2 | 24,2 | 24,4 | 24,2 | 24,3 | 24,5 | 24,8</td | | | | | | | | | | |

Rounding errors in match equity tables generating flaws in takepoints

Appendix C: Mec26 with 6 digits accuracy

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1 | 50.0000 | 68.5000 | 75.0000 | 81.8460 | 84.2500 | 89.1877 | 90.9225 | 93.6393 | 94.5936 | 96.2409 | 96.8196 | 97.7822 | 98.1205 | 98.6808 | 98.8911 | 99.2273 | 99.5454 | 99.5039 | 99.6136 | 99.7308 | 99.7719 | 99.8411 | 99.8654 | 99.9062 | 99.9205 | |
| 2 | 31.5000 | 50.0000 | 59.4518 | 66.4121 | 73.6770 | 79.4627 | 83.9040 | 86.9048 | 89.5611 | 91.8775 | 93.5001 | 94.9298 | 95.9480 | 96.5650 | 97.4890 | 98.0549 | 98.4467 | 98.7597 | 99.0442 | 99.2608 | 99.4106 | 99.5457 | 99.6381 | 99.7213 | 99.7782 | |
| 3 | 25.0000 | 40.5462 | 50.0000 | 57.1356 | 64.5982 | 70.9921 | 75.7988 | 80.0177 | 83.5346 | 86.6828 | 89.0246 | 91.1363 | 92.7226 | 94.1705 | 95.2220 | 96.1864 | 96.8855 | 97.5267 | 97.9849 | 98.4044 | 98.7033 | 98.9763 | 99.1699 | 99.3462 | 99.4710 | |
| 4 | 18.1560 | 38.5879 | 42.8644 | 50.0000 | 57.4731 | 64.0480 | 69.8653 | 73.9139 | 78.0668 | 81.734 | 84.6814 | 87.2486 | 89.3710 | 91.2255 | 92.7132 | 94.0079 | 95.0486 | 95.9540 | 96.6689 | 97.2876 | 97.7747 | 98.1952 | 98.5238 | 98.8064 | 99.0266 | |
| 5 | 15.7500 | 26.3230 | 35.4018 | 42.5269 | 50.0000 | 56.6620 | 62.2245 | 67.3423 | 71.9650 | 76.1077 | 79.5917 | 82.9909 | 85.2993 | 87.6321 | 89.5441 | 91.2456 | 92.6432 | 93.8809 | 94.8802 | 95.7601 | 96.4677 | 97.0879 | 97.5530 | 98.0145 | 98.3680 | |
| 6 | 10.8724 | 20.5373 | 29.0079 | 35.9520 | 43.3460 | 50.0000 | 55.2452 | 61.1751 | 66.1514 | 70.6255 | 74.5622 | 78.0448 | 81.1029 | 83.8197 | 86.1465 | 88.1994 | 90.9574 | 91.5038 | 92.8055 | 93.9427 | 94.8944 | 95.7223 | 96.4094 | 97.0039 | 97.4949 | |
| 7 | 9.0775 | 16.4960 | 24.0121 | 30.6347 | 37.6755 | 44.0546 | 50.0000 | 55.3358 | 60.4782 | 66.1647 | 69.4050 | 73.2138 | 76.6274 | 79.9989 | 82.3669 | 84.7896 | 86.8866 | 90.3569 | 91.7719 | 92.9799 | 94.0416 | 94.9394 | 95.7237 | 96.3534 | | |
| 8 | 6.3607 | 13.0952 | 19.9862 | 26.0861 | 32.6577 | 38.6249 | 44.8672 | 50.0000 | 55.2022 | 60.0129 | 64.4892 | 68.4951 | 72.1864 | 75.5351 | 78.5330 | 81.2281 | 83.6332 | 85.7871 | 87.6789 | 89.3391 | 90.8236 | 92.1174 | 93.2349 | 94.2168 | 95.0579 | |
| 9 | 5.4062 | 10.1389 | 16.4664 | 21.9932 | 28.0350 | 33.8486 | 39.5218 | 44.7978 | 50.0000 | 54.8172 | 59.4862 | 63.6815 | 67.5928 | 71.1900 | 74.4536 | 77.4264 | 80.1128 | 82.5459 | 84.1199 | 86.6692 | 88.3787 | 89.9136 | 91.2673 | 92.4467 | 93.4651 | |
| 10 | 3.7591 | 8.1235 | 13.3172 | 18.2866 | 23.8923 | 29.3745 | 34.9353 | 39.9811 | 45.1228 | 50.0000 | 54.6541 | 58.9288 | 63.0689 | 66.8519 | 70.3380 | 73.5938 | 75.4713 | 79.1496 | 81.5710 | 83.7625 | 85.7268 | 87.4934 | 89.0524 | 90.4636 | 91.6996 | |
| 11 | 3.1804 | 6.4989 | 10.9754 | 15.3186 | 20.4083 | 25.4378 | 30.8950 | 35.9826 | 40.5408 | 45.3459 | 50.0000 | 54.3831 | 58.5731 | 62.4926 | 66.1502 | 69.5436 | 72.6880 | 75.5868 | 78.3377 | 80.6601 | 82.2655 | 84.8486 | 86.6383 | 88.2510 | 89.6890 | |
| 12 | 2.2178 | 5.0701 | 8.8617 | 12.7514 | 17.3091 | 21.9552 | 26.7862 | 31.5049 | 36.3185 | 41.0722 | 45.6069 | 50.0000 | 54.2280 | 58.2316 | 62.0132 | 65.5527 | 69.0468 | 71.2687 | 74.0689 | 76.5669 | 79.0634 | 81.2709 | 84.0366 | 85.8523 | 87.4887 | |
| 13 | 1.8795 | 4.0520 | 7.2774 | 10.6290 | 14.7007 | 18.8871 | 23.3726 | 27.8136 | 32.4072 | 36.9301 | 41.4269 | 45.7720 | 50.0000 | 54.0145 | 57.9048 | 61.5535 | 66.0031 | 68.2387 | 71.2695 | 74.0689 | 76.5669 | 79.0634 | 81.2709 | 84.0366 | 85.8523 | 87.4887 |
| 14 | 1.3092 | 3.1420 | 5.8295 | 8.7745 | 12.3619 | 16.1803 | 20.3011 | 24.4649 | 28.8100 | 33.1481 | 37.5074 | 41.7682 | 45.9555 | 50.0000 | 53.8993 | 57.6174 | 61.1639 | 64.5183 | 67.6797 | 70.6416 | 73.4061 | 75.9780 | 78.3526 | 80.5447 | 82.5561 | |
| 15 | 1.1089 | 2.5110 | 4.7780 | 10.4569 | 13.8835 | 17.6131 | 21.4670 | 25.5464 | 29.6620 | 33.8498 | 37.9868 | 42.0952 | 46.1007 | 50.0000 | 53.7513 | 57.3604 | 60.8012 | 64.0720 | 67.1604 | 70.0870 | 72.7895 | 75.3279 | 77.8888 | 79.8886 | | |
| 16 | 0.7727 | 1.9451 | 3.8136 | 5.9921 | 8.7542 | 11.8006 | 15.2114 | 18.7719 | 22.5736 | 26.4617 | 30.4664 | 34.4473 | 38.4465 | 42.3826 | 46.2487 | 50.0000 | 53.7519 | 57.1336 | 60.4834 | 63.6702 | 66.6928 | 69.5445 | 72.2240 | 74.7316 | 77.0886 | |
| 17 | 0.6546 | 1.5533 | 3.1145 | 4.9514 | 7.3668 | 10.0426 | 13.1134 | 16.3668 | 19.8872 | 23.5267 | 27.3120 | 31.1324 | 34.9669 | 38.6361 | 42.6396 | 46.3621 | 50.0000 | 53.5228 | 56.2234 | 60.1829 | 63.2968 | 66.0556 | 69.0557 | 71.8943 | 74.1707 | |
| 18 | 0.4561 | 1.2003 | 2.4733 | 4.0460 | 6.1191 | 8.4962 | 11.2473 | 14.2129 | 17.4541 | 20.8504 | 24.4134 | 28.0500 | 31.7613 | 35.4817 | 39.1988 | 42.8862 | 46.4772 | 50.0000 | 53.1243 | 56.7315 | 59.9123 | 62.9549 | 65.8642 | 68.6042 | 71.2026 | |
| 19 | 0.3864 | 0.9578 | 2.0151 | 3.3311 | 5.1198 | 7.1945 | 9.6441 | 12.3211 | 15.2871 | 18.4250 | 21.7823 | 25.1980 | 28.7395 | 32.3203 | 36.9280 | 39.5166 | 43.0766 | 46.5747 | 50.0000 | 53.3291 | 56.5533 | 59.8577 | 62.6553 | 65.4775 | 68.1803 | |
| 20 | 0.2692 | 0.7382 | 1.5956 | 2.7124 | 4.2399 | 6.0573 | 8.2281 | 10.6409 | 13.3408 | 16.2375 | 19.3389 | 22.5722 | 25.9311 | 29.3584 | 32.8396 | 36.3298 | 38.8171 | 43.2685 | 46.2709 | 50.0000 | 53.2451 | 56.3886 | 59.4246 | 62.3383 | 65.1281 | |
| 21 | 0.2281 | 0.5894 | 1.2967 | 2.2563 | 3.5323 | 5.1056 | 7.0201 | 9.1764 | 11.6213 | 14.2732 | 17.1442 | 20.1651 | 23.3331 | 26.6939 | 29.9330 | 33.3072 | 36.0732 | 40.0877 | 43.1457 | 46.7549 | 50.0000 | 53.1639 | 56.2363 | 59.2046 | 62.0614 | |
| 22 | 0.1589 | 0.4543 | 1.0237 | 1.8048 | 2.9121 | 4.2777 | 5.9584 | 7.3682 | 10.0864 | 12.5066 | 15.1514 | 17.9629 | 20.9366 | 24.0240 | 27.2105 | 30.4555 | 33.7444 | 37.0451 | 40.3423 | 43.6104 | 46.8361 | 50.0000 | 53.0951 | 56.0937 | 59.0005 | |
| 23 | 0.1346 | 0.3619 | 0.8801 | 1.4762 | 2.4710 | 3.5906 | 5.0806 | 6.7651 | 8.7427 | 10.9316 | 13.3617 | 15.9634 | 18.7400 | 21.6474 | 24.6721 | 27.7760 | 30.9443 | 34.1458 | 37.3647 | 40.5575 | 43.7637 | 46.9095 | 50.0000 | 53.0201 | 56.9896 | |
| 24 | 0.0938 | 0.2787 | 0.6638 | 1.1936 | 1.9855 | 2.9861 | 4.2763 | 5.7513 | 7.5513 | 9.5364 | 11.7490 | 14.1477 | 16.7291 | 19.4553 | 22.3132 | 25.2684 | 28.3057 | 31.3858 | 34.5225 | 37.6607 | 40.7954 | 43.9063 | 46.9799 | 50.0000 | 52.9554 | |
| 25 | 0.0795 | 0.2218 | 0.5290 | 0.9734 | 1.6420 | 2.5051 | 3.6166 | 4.9321 | 6.5149 | 8.3004 | 10.3110 | 12.5113 | 14.9001 | 17.4439 | 20.1315 | 22.9314 | 25.8293 | 28.7974 | 31.8197 | 34.8179 | 37.9386 | 40.9995 | 44.0404 | 47.0446 | 50.0000 | |

Rounding errors in match equity tables generating flaws in takepoints

Appendix D: Dead Cube Takepoints derived from Mec26 with 6 digits accuracy

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|----|---|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | |
| 2 | x | 31,5 | 36,5 | 36,3 | 38,8 | 32,2 | 34,5 | 31,0 | 32,8 | 28,7 | 30,5 | 27,3 | 28,9 | 25,8 | 27,4 | 24,6 | 26,1 | 23,4 | 24,9 | 22,4 | 23,8 | 21,5 | 22,8 | 20,6 | 22,0 |
| 3 | x | 27,0 | 31,1 | 37,4 | 30,9 | 30,3 | 28,2 | 29,7 | 26,8 | 26,9 | 25,0 | 25,7 | 23,9 | 24,3 | 22,7 | 23,2 | 21,8 | 22,2 | 20,9 | 21,3 | 20,2 | 20,6 | 19,6 | 19,9 | 19,0 |
| 4 | x | 18,9 | 22,9 | 28,3 | 29,5 | 28,8 | 29,6 | 30,1 | 29,8 | 28,9 | 29,0 | 28,7 | 28,6 | 28,0 | 27,9 | 27,4 | 27,2 | 26,6 | 26,5 | 25,9 | 25,8 | 25,2 | 25,1 | 24,6 | 24,5 |
| 5 | x | 17,2 | 20,8 | 23,2 | 24,4 | 24,4 | 24,9 | 26,1 | 25,8 | 25,7 | 25,5 | 25,9 | 25,5 | 25,7 | 25,3 | 25,5 | 25,0 | 25,0 | 24,6 | 24,7 | 24,3 | 24,3 | 23,9 | 23,9 | 23,5 |
| 6 | x | 21,6 | 23,3 | 25,4 | 26,3 | 26,3 | 27,1 | 27,5 | 27,6 | 27,3 | 27,5 | 27,4 | 27,5 | 27,3 | 27,1 | 27,0 | 26,7 | 26,7 | 26,4 | 26,3 | 26,0 | 25,9 | 25,7 | 25,5 | |
| 7 | x | 22,0 | 24,3 | 25,3 | 25,8 | 25,6 | 25,9 | 26,5 | 26,5 | 26,4 | 26,3 | 26,5 | 26,4 | 26,4 | 26,3 | 26,4 | 26,2 | 26,1 | 26,0 | 25,9 | 25,7 | 25,7 | 25,5 | 25,4 | 25,2 |
| 8 | x | 19,7 | 21,2 | 23,3 | 24,3 | 24,9 | 25,5 | 26,2 | 26,4 | 26,5 | 26,6 | 26,8 | 26,9 | 26,9 | 26,9 | 26,9 | 26,8 | 26,7 | 26,6 | 26,5 | 26,4 | 26,3 | 26,2 | 26,1 | |
| 9 | x | 20,6 | 22,4 | 22,5 | 23,7 | 23,6 | 24,2 | 24,8 | 25,2 | 25,3 | 25,5 | 25,7 | 25,9 | 26,0 | 26,1 | 26,2 | 26,2 | 26,2 | 26,1 | 26,2 | 26,1 | 26,1 | 26,0 | 25,9 | |
| 10 | x | 20,3 | 21,7 | 23,1 | 23,9 | 24,2 | 24,8 | 25,2 | 25,5 | 25,6 | 25,9 | 26,0 | 26,2 | 26,3 | 26,4 | 26,4 | 26,5 | 26,5 | 26,4 | 26,4 | 26,4 | 26,3 | 26,3 | 26,2 | |
| 11 | x | 22,2 | 23,7 | 23,6 | 24,3 | 24,1 | 24,5 | 24,8 | 25,1 | 25,2 | 25,4 | 25,6 | 25,7 | 25,8 | 25,9 | 26,0 | 26,1 | 26,1 | 26,1 | 26,2 | 26,2 | 26,2 | 26,1 | 26,1 | 26,1 |
| 12 | x | 20,0 | 21,1 | 22,5 | 23,2 | 23,7 | 24,2 | 24,7 | 25,0 | 25,2 | 25,4 | 25,6 | 25,8 | 25,9 | 26,0 | 26,1 | 26,2 | 26,2 | 26,2 | 26,3 | 26,3 | 26,3 | 26,3 | 26,3 | 26,2 |
| 13 | x | 22,0 | 23,2 | 22,8 | 23,6 | 23,5 | 24,0 | 24,2 | 24,6 | 24,7 | 25,0 | 25,2 | 25,3 | 25,5 | 25,6 | 25,7 | 25,8 | 25,9 | 26,0 | 26,0 | 26,1 | 26,1 | 26,1 | 26,1 | 26,1 |
| 14 | x | 20,1 | 21,0 | 22,1 | 22,7 | 23,2 | 23,7 | 24,1 | 24,4 | 24,6 | 24,9 | 25,1 | 25,3 | 25,4 | 25,6 | 25,7 | 25,8 | 25,9 | 26,0 | 26,0 | 26,1 | 26,1 | 26,1 | 26,1 | 26,1 |
| 15 | x | 22,5 | 23,5 | 22,8 | 23,5 | 23,4 | 23,8 | 24,0 | 24,3 | 24,5 | 24,7 | 24,9 | 25,0 | 25,2 | 25,3 | 25,5 | 25,6 | 25,7 | 25,8 | 25,8 | 25,9 | 25,9 | 26,0 | 26,0 | 26,0 |
| 16 | x | 20,1 | 20,8 | 21,8 | 22,4 | 22,8 | 23,3 | 23,7 | 24,0 | 24,2 | 24,5 | 24,7 | 24,9 | 25,1 | 25,2 | 25,4 | 25,5 | 25,6 | 25,7 | 25,8 | 25,8 | 25,9 | 25,9 | 26,0 | 26,0 |
| 17 | x | 22,5 | 23,4 | 22,6 | 23,2 | 23,1 | 23,4 | 23,6 | 23,9 | 24,1 | 24,3 | 24,5 | 24,7 | 24,8 | 25,0 | 25,2 | 25,3 | 25,4 | 25,5 | 25,6 | 25,6 | 25,7 | 25,7 | 25,8 | 25,9 |
| 18 | x | 20,1 | 20,8 | 21,7 | 22,2 | 22,7 | 23,1 | 23,4 | 23,7 | 23,9 | 24,2 | 24,4 | 24,6 | 24,7 | 24,9 | 25,1 | 25,2 | 25,3 | 25,4 | 25,5 | 25,6 | 25,6 | 25,7 | 25,7 | 25,8 |
| 19 | x | 22,7 | 23,5 | 22,7 | 23,2 | 23,0 | 23,3 | 23,5 | 23,8 | 23,9 | 24,1 | 24,3 | 24,4 | 24,6 | 24,8 | 24,9 | 25,0 | 25,2 | 25,3 | 25,4 | 25,5 | 25,5 | 25,6 | 25,7 | 25,7 |
| 20 | x | 20,2 | 20,8 | 21,6 | 22,1 | 22,5 | 22,9 | 23,2 | 23,5 | 23,7 | 23,9 | 24,1 | 24,3 | 24,5 | 24,7 | 24,8 | 24,9 | 25,1 | 25,2 | 25,3 | 25,4 | 25,5 | 25,5 | 25,6 | 25,7 |
| 21 | x | 22,8 | 23,5 | 22,6 | 23,0 | 22,9 | 23,2 | 23,3 | 23,5 | 23,7 | 23,9 | 24,0 | 24,2 | 24,4 | 24,5 | 24,7 | 24,8 | 24,9 | 25,0 | 25,1 | 25,2 | 25,3 | 25,4 | 25,5 | 25,6 |
| 22 | x | 20,3 | 20,8 | 21,6 | 22,0 | 22,4 | 22,7 | 23,0 | 23,3 | 23,5 | 23,7 | 23,9 | 24,1 | 24,3 | 24,4 | 24,6 | 24,7 | 24,8 | 24,9 | 25,0 | 25,2 | 25,2 | 25,3 | 25,4 | 25,5 |
| 23 | x | 22,9 | 23,5 | 22,6 | 23,0 | 22,8 | 23,1 | 23,2 | 23,4 | 23,5 | 23,7 | 23,9 | 24,0 | 24,2 | 24,3 | 24,5 | 24,6 | 24,7 | 24,8 | 24,9 | 25,0 | 25,1 | 25,2 | 25,3 | 25,4 |
| 24 | x | 20,3 | 20,8 | 21,5 | 21,9 | 22,3 | 22,6 | 22,9 | 23,1 | 23,3 | 23,5 | 23,7 | 23,9 | 24,1 | 24,2 | 24,4 | 24,5 | 24,6 | 24,7 | 24,8 | 25,0 | 25,0 | 25,1 | 25,2 | 25,3 |
| 25 | x | 23,0 | 23,5 | 22,5 | 22,9 | 22,7 | 22,9 | 23,0 | 23,2 | 23,4 | 23,6 | 23,7 | 23,9 | 24,0 | 24,1 | 24,3 | 24,4 | 24,5 | 24,6 | 24,7 | 24,9 | 24,9 | 25,0 | 25,1 | 25,2 |