Rough and Ready Rules for Rookies: The First Two Rolls

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Abstract

You are an ambitious backgammon player? You play opening rolls depending on the mood of the day in a couple of ways? You have heard about splitting, slotting, and playing "two down"? You know that hitting is very important, but so is making points and running to safety? You are unsure what to do with a reply roll that can accomplish only one of these things? You are likewise struggling with doublets that offer plenty of good choices? You sometimes play from the rear and sometimes from the mid-point after hitting, but have no idea which is better? You like to hit twice in your home board and sometimes feel that this is fun, but too aggressive? You know that there are large tables with correct reply moves out there, but do not want to learn 630 of them by heart? You are willing to sacrifice some accuracy for getting a smaller number of rules of thumb that you can actually memorize and thus will use?

Then you should read this article.



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1 Introduction

Backgammon is fast-paced. A game is over quickly, so again and again we need to play opening or reply moves. These occur so frequently that it pays to learn a little about the first two rolls. Why not three? Because the dice rolls lead to a "combinatorial explosion": after just the third roll, a player is confronted with one of about 25,000 different situations. Only chess players and medical students might be willing to try to commit this to memory.

The first move, the opening, is trivial. Because no doublets can occur, only 15 moves need to be learned for perfect play. But since in real life many rolls will be played in reasonable, but *non-perfect* ways, we will encounter one of about 30 different positions before the shake for the second move. Multiplied by 21 possible reply rolls (now *including* doublets), this results in one of 630 different situations.

Fortunately, there is a great article by world class player JEREMY BAGAI, who wrote in Bagai's Replies: Mastering the Second Roll how to reply almost perfectly. Unfortunately, his 12 rules, 26 sub-rules, and 26 exceptions are still too numerous for my memory and unnecessarily accurate for my level of play (I have much bigger problems to tackle). So I boiled his method down to something palatable for players of at most low advanced level. I certainly sacrificed a lot of his accuracy for (hopefully) a simpler set of rules with fewer exceptions. If you want to go for the real thing, though, click the link above (it is really a trailblazing work, I cannot recommend it highly enough).

My approach to this is as follows: First I will outline the **notation** I am using that makes it easier for *you* to understand and for *me* to write this article. To get familiar with this, at a comfortable pace, let us begin with the **opening** moves. This will still be pretty easy.



Figure 1: Beautiful swan (By Diego Delso, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=18759800)

It becomes more complicated when we get to the non-doublet reply moves. This is the main part of the article, and it is written as a mirrored version of the fairy tale "The Ugly Duckling" that could be named "The Ugly Cygnet". In my story, Bagai's rule set is the "Beautiful Swan", very nice, but (at least for me) not edible and too big a bite. So I will have a closer look at the "Beautiful Cygnet", an extremely stripped-down initial rule set, derived from BAGAI's, which will then be tested. Next, I will smuggle a duck's egg into the swan's nest, a slightly different, but still small rule set, derived from my musings on opening moves. Unfortunately, that "Ugly Cygnet" is clumsy and the test results will further stigmatize it as inept. Fortunately, this will prove very useful for my next step: by listing and grouping the blemishes of this rule set, I will be able to augment it with further rules and thus refine it, producing my final rule set called "Pretty Duck". This species of waterfowl is ready to dive into action across the board (including doublet reply moves) being lightweight, robust, and workable. It is smaller and less graceful than the "Beautiful Swan", but in my opinion, it has a significant advantage: it is edible.

This is my promise: You will be able to play the

- 1. opening moves in money games¹ without any errors²,
- 2. opening moves for all possible scores of a 5-point match without any semi-whoppers³, and
- 3. reply moves in money games without any semi-whoppers.

All this is based on the most accurate available data from eXtreme Gammon's Opening book and replies page (in short: "XG database") and presented within a very simple and easy to memorize framework.

Enjoy your meal!

But in case you are not interested in the long, long train of thought that leads to the necessary additions to the rule set, to progress from the "Ugly Cygnet" to the "Pretty Duck" (trust me, I learned a lot!), you should first read about the notation before feeling free to jump directly forward to the summary. If you are even less ambitious, reading only notation, opening, and "Beautiful Cygnet" could be sufficient for beginners and casual players. However, this neither keeps the third promise above nor covers doublets⁴). In contrast, if you are more interested in the technical details that plagued me during the research for and the writing of this article, you should read the footnotes, which are often used to tuck away stuff that might otherwise confuse casual players. The footnotes (as well as the appendix) are more geared towards semi-experts, in case this article attracts such readers.

2 Notation

This article uses a short notation system invented by world class player NACK BALLARD for (mostly) the early stages of a backgammon game. All we need to know for our purpose can be found in the following table:

Nactation	Definition
P	Point. Make a point, or point on a blot.
R	Run. Move a back checker to the outfield.
U	Up. Advance back checker(s), up to and including the opponent's
	bar point.
	Exception: 21U means 24/21, not 24/23 24/22.
D	Down. Play from the mid-point to the outer board.
S	Split. Play the larger number with a back checker, and play the
	smaller number down.
Z	Reverse split. Play the smaller number with a back checker, and
	play the larger number down.
\$	Slot. Play a checker down and put it or another checker onto a
	vacant offensive point.
W	Wild. Split and slot in the same move.
Η	Hit. Hit an opposing checker, and play the other number down.
X	Hit and split. Hit on the near side with one number, and play on
	the far side with the other number.
K	Kill. Hit two checkers.

It is a full-blown system, though, and the interested reader can find the documentation of over 120 pages in Nactation.

Using Nactation, play can be described concisely and accurately. 54D-41S means our opponent rolled a 54 and played two checkers down from her midpoint $(13/9 \ 13/8)$. Then we rolled a 41, split our back checkers and brought a builder down from our mid-point $(24/23 \ 13/9)$:

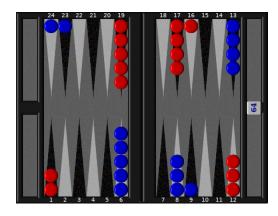


Figure 2: 54D-41S: Lost 81 millipoints

If you are new to Nactation and have been paying attention, you may ask: how can you play the smaller number of a 41 roll down from the mid-point? The answer is that Nactation has a "convenience clause": When it is impossible to play the small number down to the near side, "S" can be used to represent a reverse split ("Z"). The same holds if we cannot play the larger number on the far side, like 54S. This notation is very common and virtually the default in discussions about opening and reply moves.

Nactation can also be handy for comparing various plays using Nacbracs (Nactation brackets): For example, the Nacbrac

54D-41 [\$ W57 S81]

for the situation we discussed above means that the best move is to play the 4 down and slot point 5 with the 1 (\$). The next best is the wild "splotting" play to split with the 4 and slot with the 1 (W). This move, however, already loses 0.057 of equity or 57 millipoints. The final entry in the Nacbrac means that our choice of splitting (24/23 13/9 in this case) loses 81 millipoints, as shown in the previous figure. So in this example, only the slotting play would avoid a semi-whopper.

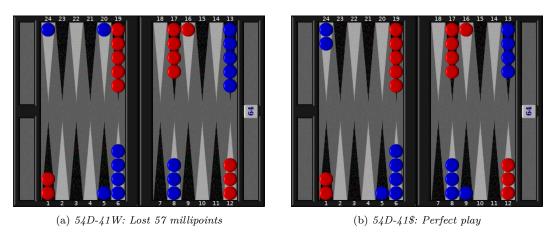


Figure 3: Moves for 54D-41

3 Opening

The opening moves have been studied for decades. Over time, fashions have come (backed up by a lot of gut feeling masked as sound theory) and gone (backed up by "irrefutable" computer analysis). But the programs became stronger, so only a few years later the "hard facts" had to be revised.

Here is a nice quote by Walter Trice from his great book "Backgammon Boot Camp":

Most of robotic opening theory can be summed up in fewer than twenty words: Run with 65 and 64; make a point if you can; otherwise, never slot and always split.

That was 2004. With the help of Nactation, we can summarize the *present* state of the art on opening moves as:

This translates as: "Run with 65; split with 64; make a point if you can; play two down with 43; slot with 21; otherwise split." These are the "current hard facts" for money games (first promise), and we can see that over the course of 9 years (the XG database is from 2013), 3 moves out of 15 have changed again.

Opening moves do not only change over time, but also during a match, because they depend on the score: the best moves for a particular roll at Gammon-Go, Gammon-Save, Double Match Point and CRAWFORD games can be quite different. As a lazy chap, I was not willing to memorize dozens of chains of Nactation symbols for all situations in match play, but was willing to sacrifice some accuracy for simplification. After some searching, I found a list⁵ of opening moves that manages to avoid semi-whoppers⁶ for all 33 possible match scores of a 5-point match (second promise):

This is similar, but not equal to the best moves for money games: for match play 32D, 51\$, and 41\$ replace the corresponding splits.

However, I strongly recommend using only this single latter list for *both* match play and money games. The reason is that the correct reply moves against D or \$ are sometimes non-intuitive (e.g., many opponents, especially up to intermediate level, will not dare to slot).⁷ To accept a small equity loss⁸ by playing an opening mistake will thus hopefully induce a much larger reply error by our opponent and easily compensate us.

If you follow my advice, we are all set with the topic of opening moves: "One list to learn it all, one list to roll them, one list to move them all, and in backgammon beat them." O. K., maybe not quite.



Figure 4: Beautiful cygnet (By NoRud, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=37994507)

4 Reply (non-doublets)

4.1 Starting out – The "Beautiful Cygnet"

4.1.1 Candidate rule set

BAGAI mentions in his article that his 6 main rules for non-doublet replies are "a great way to introduce the replies to a beginner". Indeed they are, but on the one hand they do not mention what to do with the remainder of the roll after hitting (this is specified by means of sub-rules) and on the other hand they still contain qualifications (e. g., based on how the opening was played). For the former, I will employ some of his sub-rules; for the latter, I will dispense with all special cases and opt for the most common variant. Finally, we need alternative moves if our opponent has made a point which blocks our default move. After reshuffling a little, we arrive at our initial rule set:

- 1. Hit above 6
- 2. 65R, P
- 3. Hit above 3
- 4. 5 on mid-point? D, else U
- 5. 43Z, S

(If blocked: 65D, 63R, 62R, 54D, 52D, 43S, 32Z, 51\$, 41\$, 21\$)

What may look a little cryptic, is in fact the recipe for a great duck stew (meaning for ducks, not from ducks). Let us have a look at all the ingredients in turn.

4.1.2 Hit above 6 (Hitting outside)

We should **hit** as many blots as possible outside our home board (i. e., **higher than** our point **6**). In case several moves result in the same number of hits, we should prefer the one that hits on the higher points, thus maximizing our gain in the race.

4.1.3 65R, P (Defaults for good rolls)

If we could not hit any outside blot and had rolled **65**, it is now time to **Run**. If we can make any **Point** (with 31, 42, 53, 64, and 61), we should do so. This avoids waste in the kitchen by eating our (bread) rolls now instead of letting them get stale.

4.1.4 Hit above 3 (Hitting inside)

Unless we replied by hitting twice, a pointing roll, or 65R and thus have completed our move, there may still be blots up for grabs in our home board. If so, we should **hit** as many of these as high as possible, as before. But we should not get into a blood frenzy: hitting deeper than point 4 is often "too deep", hence this rule restricts us to points **higher than** our point **3**. There will be opportunities for more hitting later, little cygnet, I promise. Wait till you are big and strong.

4.1.5 5 on mid-point? D, else U (After hitting)

This part applies *only* when we have hit exactly one blot and specifies how to play the remainder of the roll. With our checkers in the starting position,

we might hit from points 24, 13, 8, or 6. After hitting from points 24, 8, or 6, we would still have **5** checkers **on** the **mid-point** (our point 13). So we should unstack it by using the remaining number to move one checker **Down** from the mid-point.

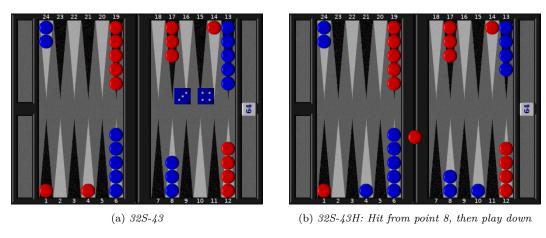


Figure 5: After hitting: Unstacking

However, if we hit from the mid-point, we already accomplish some unstacking, so should use the second part of the roll for something **else**, i. e., splitting by moving one checker **Up** from the rear (our point 24).

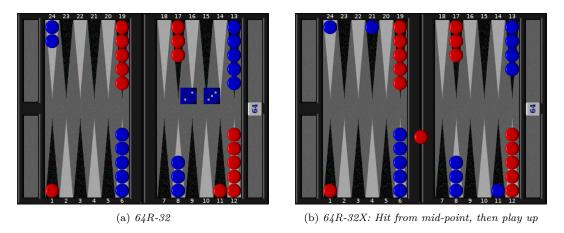


Figure 6: After hitting: Splitting

What if D or U is blocked? Simple, "DUUD". If D is blocked, play U. If U is blocked, play D:

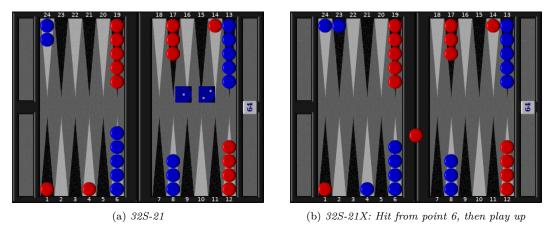


Figure 7: After hitting: 13/12 is blocked.

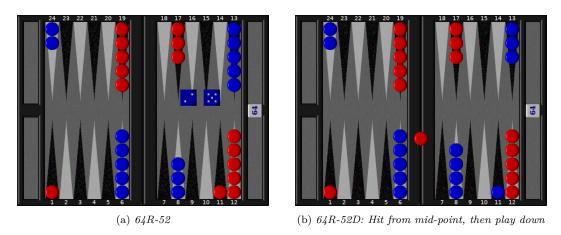


Figure 8: After hitting: 24/19 is blocked.

4.1.6 43Z, S (Defaults for other rolls)

This step will cover the other rolls in case we could not hit. If we rolled **43**, we should play the reverse split (**Zplit**, see Nactation table for a refresher), which translates into standard backgammon notation as "24/21 13/9". For all the other leftover rolls (21, 41, 51, 62, 63, 32, 52, 54), we should play the standard **Split** (keeping in mind the "convenience clause").

4.1.7 Results

Now let us test the waters with our "Beautiful Cygnet". It has to swim through all 450 non-doublet reply situations (I will deal with doublets in a separate

section). For each, I will compare the correct move from the XG database with the move we get by adhering to our set of a mere 5 rules. Our young talent will then be measured and judged by all kinds of numbers, here they are:

Criterion	Result
Correct moves	331 (73%)
$Mistakes \leq 0.05$	98(22%)
Mistakes > 0.05	21 (5%)
Total equity loss	3.265
Maximum equity loss per mistake	0.100
Average equity loss per mistake	0.027
Average equity loss per move	0.007

For such a simplistic rule set, playing without whoppers is really impressive. Almost 3 out of 4 moves are played perfectly. If you are already content with this, feel free to stop reading this article immediately and leave for the next online competition. However, my promise was to teach you to play without semi-whoppers in order to safeguard you from embarassment by the other big birds found at tournaments. Between whoppers and semi-whoppers there is still a factor of 2, requiring a further reduction in the size of mistakes (reducing the number of mistakes would of course also be welcome). In case you want to breed some waterfowl yourself, you can find the Nacbracs for all mistakes in Mistakes by "Beautiful Cygnet". These should enable you to experiment with alternative rule sets.

One interesting aspect of these results relates to my recommendation to play bolder opening moves like D or \$, even if technically they may be mistakes. Against these, the timid splitting reply moves frequently preferred by casual players (and our "Beautiful Cygnet") lose almost 1000 millipoints of equity. This is almost one third of the total equity loss in the above table! These safe plays cause 33 mistakes (5 of them semi-whoppers, the largest losing 81 millipoints). So my hope of inducing larger reply errors from opponents is backed up by the data. This also suggests how the behavior of our "Beautiful Cygnet" might be improved further in the next section.



Figure 9: Duck's egg (By Judgefloro, CCO, https://commons.wikimedia.org/w/index.php?curid=61542142)

4.2 Smuggling in – The "Ugly Cygnet"

4.2.1 Genome editing of waterfowl

First, let us take a closer look at the "Beautiful Cygnet" in order to consider what improvements an "Ugly Cygnet" (smuggled in as a duck's egg) requires to become more competitive. Some structural features have already been hinted at, see the headings in the previous section:

- 1. Hitting
- 2. After hitting
- 3. No hitting (default moves)

Next, consider two facts for the 450 non-doublet replies from the XG database of perfect play: $161~(36\,\%)$ of the cases involve hitting, and in $202~(70\,\%)$ of

the 289 non-hitting cases our default moves were correct. That leaves 87 cases in which our default moves were incorrect.⁹

Perhaps we can reduce these mistakes (in number and size) by using other default moves, such as the ones suggested in Opening. Re-using them as reply moves might be a good default. After all, not much has changed since our opponent played her opening move. But has it? A sneak preview of the XG database reveals that 64S is rarely a good reply, 64P would be better. However, we need 64S as an opening move in matches. As a quick fix for now, let me just define 64S as default for opening moves and 64P as default for reply moves.

With this out of the way, let us see whether the "Ugly Cygnet" performs better in our test water with one of its five genes (the "Defaults for other rolls") modified by CRISPR-Cas9.



Figure 10: Ugly cygnet (By Nevit Dilmen, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=1393584)

4.2.2 Candidate rule set

- 1. Hit above 6
- 2. 65R, P
- 3. Hit above 3
- 4. 5 on mid-point? D, else U
- 5. 43D, 32D, 51\$, 41\$, 21\$, S

(If blocked: 65D, 63R, 62R, 54D 52D)

4.2.3 Results

Criterion	Result
Correct moves	325 (72 %)
$Mistakes \leq 0.05$	89 (20%)
Mistakes > 0.05	$36 \ (8 \%)$
Total equity loss	4.579
Maximum equity loss per mistake	0.124
Average equity loss per mistake	0.037
Average equity loss per move	0.010
Maximum equity loss per mistake Average equity loss per mistake	0.124 0.037

These results are, in a word, terrible. Our "Ugly Cygnet" performs worse on all important accounts: it makes more mistakes (especially large ones), and suffers more equity loss (total and maximum). Time to commit suicide? Not yet, my young swan. Perhaps we should reflect on what went wrong with the genome editing and how to correct this. Time to dive into the Nacbracs, dabble around among the large mistakes and see what can be dredged up. The Nacbracs for all mistakes can be found in Mistakes by "Ugly Cygnet".



Figure 11: Pretty duck (By Bert de Tilly, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=47203056)

4.3 Stepping up – The "Pretty Duck"

4.3.1 Introduction

The results from the previous section put 36 mistakes on our to-do list (or rather not-to-do list): 33 semi-whoppers and 3 whoppers need to be avoided to claim replies free of semi-whoppers. We could of course work through the list starting with the largest mistakes, but a nice feature of Nacbracs is that they easily allow sorting not only by opening and reply roll, but also by opening and reply $action^{12}$. So the mistakes will be grouped according to our general outline of a rule set, and we will proceed along this way instead.

Claiming victory by quickly adding 36 specific exceptions to our set of 5 general rules would not be very smart, because there *is* structure hidden in the data that allows for a *better and smaller* rule set with *fewer* exceptions. General rules (in contrast to specific exceptions) can have five effects:

- They fix *several* semi-whoppers (our intention).
- They fix smaller mistakes (that were not on our to-do list).

- They cause smaller mistakes (that need to be balanced with the cases fixed).
- They cause semi-whoppers (that need to be treated with further rules or exceptions).
- They might be good guidelines not only for the first two rolls, but broadly speaking for the whole opening stage of a game.

The next sections will present a lot of findings from the reply data. We will examine small groups of typical semi-whoppers in turn. Then we will try to make sense from their Nacbracs. For this, it helps to know that the *first* entry in the Nacbrac (i. e., the one after the opening "[") is the perfect play (zero equity loss). Each subsequent entry has a number attached, denoting the equity loss for that play in millipoints. Of special interest to us is the *last* entry, which is the mistake that our "Ugly Cygnet" has made (I have truncated the longer Nacbracs at this point accordingly). One random pick from the appendix may help to brush up our Nactation knowledge:

```
63S-43 [Z S6 U35 H55 ... D85]
```

This shows that following an opening roll of 63 (played by the opponent as **Split**, 24/18 13/10), our "Ugly Cygnet" rolled 43 and moved two checkers **Down** from the mid-point. This move lost 85 millipoints (0.085 of equity). The perfect play was the "reverse split" (**Zplit**), 24/21 13/9. (The ellipsis "..." indicates that I have omitted unneeded or unknown data from the rollout.)

You may try to understand the other letters in this Nacbrac and find out how the roll was played in each instance. Now if we look long enough at many such lines of letters, numbers and symbols, we might begin to hatch ideas for additional rules and alternative moves to help our "Ugly Cygnet" mature into a "Pretty Duck". I will help you with this project.

4.3.2 Hitting deep (with non-pointing rolls)

Our first group of Nacbracs has two whoppers and a semi-whopper to dabble with. Here, we played the default according to rule 5, two checkers down from the mid-point:

```
32Z-32 [K X4 H11 S62 Z74 D90]
32Z-43 [X H11 ... Z93 S97 D124]
52S-43 [H X3 U46 S63 ... D108]
```

Note that, no matter how the remainder of the roll is played, all rolls contained

a 3 that should have been used to hit on point 3. This is deeper than stated by our rule 3 and suggests a very modest addition to our "Pretty Duck": ¹³

• With 43 or 32: 6/3*

While I promised you more blood in Hit above 3 (Hitting inside), this last addition delivers only a half-hearted license to hit on one particular point. It will become better now: James Bond is notorious for having a license to kill, but I will grant you two!

An interesting observation I have made is that many beginners are afraid to slot or aggressively hit on points 5 and 4, but will happily kill (hit twice) deep in their home board. The former is very often right; the latter often wrong. Have a look at the Nacbracs:

```
21S-41 [K S35 U47 X48 D52 H84 $95]
41S-41 [K S47 X52 U57 D59 $82]
51S-41 [K D15 U31 S51 X52 W77 $78]
62S-65 [K D50]
63S-65 [K D60]
64S-65 [K D75]
```

These six cases are written in standard backgammon notation as $6/2^*/1^*$ and $13/7^*$ $6/1^*$, respectively.

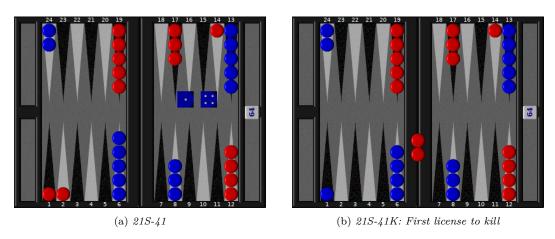
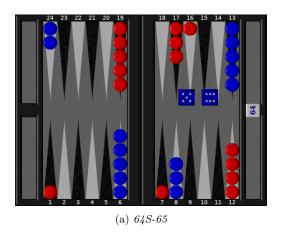
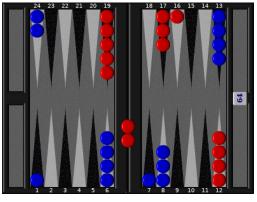


Figure 12: Hitting deep: Killing





(b) 64S-65K: Second license to kill

Figure 13: Hitting deep: Killing

They are the *only* cases not yet covered by our normal rules for hitting where killing is correct.¹⁴ To repeat, this means that the other killing moves that are often played (because it is fun?) are *wrong*.

So our "Pretty Duck" will include the following killing rules:

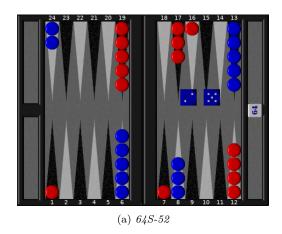
• With 65: 13/7* 6/1*

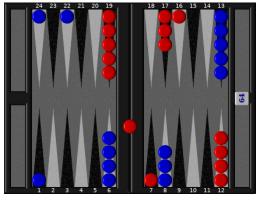
• With 41: 6/2*/1*

The last two semi-whoppers in this section are:

64S-52 [X S56] 64S-54 [X S56]

There are in total five cases in which the opponent splits to her bar-point and perfect play requires us to hit deep on point 1 with 52 (or 54) and to split the rear checkers ("X" in Nactation).





(b) 64S-52X: Checker on bar-point now helpless

Figure 14: Hitting deep: Communication breakdown

This gives us our last rule for hitting deep: 15

• With 54 or 52: After 62S or 64S, play $24/x 6/1^*$

4.3.3 Hitting high (with pointing rolls)

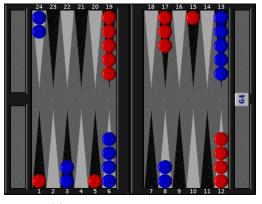
The Nacbracs reveal one semi-whopper where perfect play demands hitting on point 5, but we made a point with 53P according to rule 2:

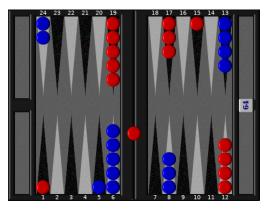
There are two semi-whoppers where perfect play demands making point 5 with 31P, but we hit on the bar-point according to rule 1:

```
62S-31 [P X40 H53]
63S-31 [P X46 H60]
```

Note that all cases involved rolls for which a potential point was *deeper* on our side of the board than a juicy blot. In the two latter cases, "pointing deep" was correct; in the first case "hitting high" was right. Obviously, we have to strike (pun intended) the proper balance here. How deep is too deep?

By serendipity I found a strange coincidence: a good limit for pointing rolls can be calculated by subtracting the pips rolled from 12. Let us call this the "Rule of 12". We should hit on points *higher* than this limit. For example, with 31 we should hit above point 8, with 61 above point 5, and with 64 above point 2. Whether we hit or not *above* this point, we then try to point on head ("POH").¹⁶ This will handle all three cases above correctly.¹⁷





(a) 43S-53P: Lost 75 millipoints

(b) 43S-53H: Perfect play

Figure 15: Hitting high: Rule of 12

So our "Pretty Duck" will include our "Rule of 12" for pointing rolls:

• Hit above "12 - roll", then point on head

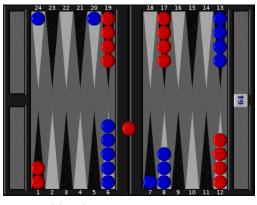
4.3.4 After hitting

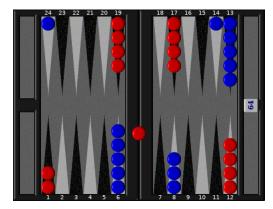
There is even less work to do here. Rule 4, which specifies how to play the remainder of the roll after exactly one blot has been hit, is just extremely accurate. Of course it was *not* invented here, but to my knowledge is a brain child of Jeremy Bagai. Did I already recommend reading his article? I think so.

The three relevant cases that "5 on mid-point? D, else U" gets wrong (but typically every beginner gets right) are the following: 19

41\$-64 [R Z57] 51\$-64 [R Z53] 62\$-64 [R Z50]

So after hitting the blot on our "Golden Point", we should continue and run into the outfield, in standard backgammon notation: 24/20*/14. That's it.





(a) 51\$-64Z: Lost 53 millipoints

(b) 51\$-64R: Perfect play

Figure 16: After hitting: 20/14

So our "Pretty Duck" will include the following "after hitting" rules (rewritten here without the use of Nactation):

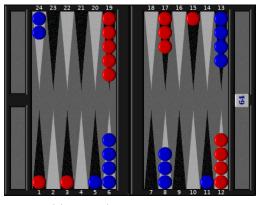
- 20/14
- 5 checkers on your mid-point? Play 13/x to unstack, else 24/x

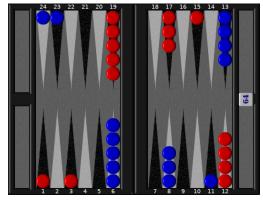
4.3.5 Default moves after opponent splits

We have covered already a lot of ground in the last sections. So far we had no reason to think about our default moves: they were not needed yet, because our move was completely governed by the rules *for* and *after* hitting. This will change now. We will start with the following Nacbracs for a couple of semi-whoppers:

```
32Z-21 [S H4 V45 U54 D71 $75]
21S-51 [S U48 X62 $76]
```

Perfect play demands splitting our back checkers with the 1 and playing the other number down from the mid-point (21S, 51S), but we played the default slots 21\$ and 51\$ according to rule 5. Slotting is a fine move, but not if our opponent has split her back checkers, because we leave her a double shot. Likewise, we do not break into a house if we know that the owner is awake, armed, and has a fierce dog running around.





(a) 32Z-21\$: Lost 75 millipoints

(b) 32Z-21S: Perfect play

Figure 17: Default moves: No \$ after S/Z

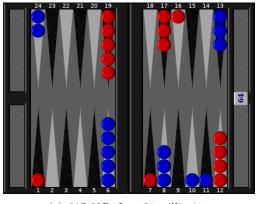
In these cases and ten similar ones, including five semi-whoppers, the correct move is always to split (S). The rule to trigger the use of these alternative moves is also simple:

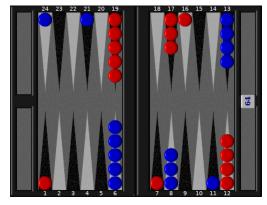
• Play S after opponent has split (S/Z)

There is another class of semi-whoppers (seven in total), here are some typical Nacbracs:

```
64S-32 [S H18 Z30 ... D84]
62S-43 [Z U10 S41 ... D87]
```

Perfect play demands split (S) and reverse split (Z) here, but we played two checkers from the mid-point (D), leaving two direct shots in the outer board for the opponent's checker waiting on our bar-point.





(a) 64S-32D: Lost 84 millipoints

(b) 64S-32S: Perfect play

Figure 18: Default moves: No D after S

The remedy is simply to slightly augment our first addition from this section:

• Play 43Z or S after opponent has split (S/Z)

The extended rule automatically includes the previous cases with double shots in our home board, resulting from inappropriate slots. This is great, but we need to ensure that we do not play 31S or 65S by accident. So a careful ordering of our rule set or proper restrictions on candidate moves are required.

The wording of this rule is quite specific to the opening stage. If you have bought into my reasoning for generic rules and thus prefer a different phrasing,

• Leave no double shot on near side

will give the same results, but might prove more useful later on in the game.

4.3.6 Default moves after opponent makes a point

The next class of semi-whoppers is similar to the first in the previous section:

31P-21 [S V38 U53 \$65]

31P-41 [S D57 \$83]

42P-51 [S \$57]

Again the slot is wrong and the split is right, but this time the reason is different: although our opponent has not split her back checkers, she has strengthened her home board by making a point, so slotting becomes more hazardous. It is a bad idea, like breaking into a house when the legal penalty for burglary has increased considerably.

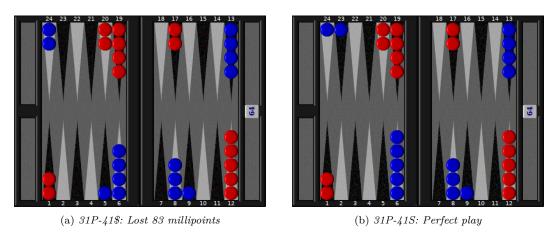


Figure 19: Default moves: No \$ after P

The concept can be generalized to other bold default replies:

61P-32 [Z S1 U23 D41] 31P-43 [Z D28]

Here, playing two checkers down from the mid-point is wrong and again the split is right. These mistakes are below semi-whopper size, because two indirect shots are much less risky than the two direct shots we left in the previous section.

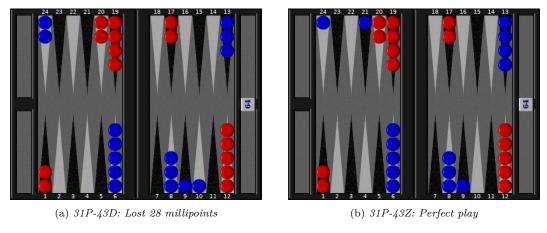


Figure 20: Default moves: No D after P

The rule to trigger the use of these alternative moves in 17 cases (4 semi-whoppers) is as follows:

• Play 43Z or S after opponent has made a point (P)

(If you prefer a more generic wording, then

• Play safe after opponent has strengthened her home board

will do. It also nicely echoes Magriel's Safe-Bold Criteria, which are extremely useful concepts throughout a game.)

Again, the same proviso as in the previous section applies with respect to rule set ordering and candidate moves. Also note that we can now merge this rule with the one from the previous section:

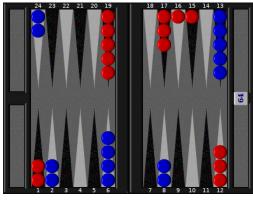
• Play 43Z or S after opponent has split (S/Z) or made a point (P)

4.3.7 Default moves after opponent brings builders

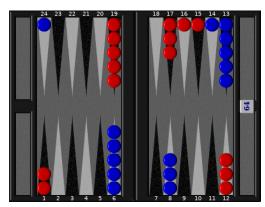
The next semi-whopper made by "Ugly Cygnet" is this:

43D-64 [R S1 P71]

Although this is just a single case, it will pay us a lot to dive into it deeper. What has happened? Our opponent used her 43 opening roll to bring two checkers down from her mid-point. We made point 2 with our 64 reply. How bad can it be to have an additional home board point? It should deter our opponent from playing too aggressively with her now numerous builders. But this move loses 71 millipoints? Key to understanding this is that while making point 2 is usually good, as a reply to this opening move there is a far superior alternative: "Get the hell out of there!"







(b) 43D-64R: Perfect play

Figure 21: Default moves: Running

It is important to note that 64R manages to run past *all* builders that our opponent brought from the mid-point into her outer board. A more detailed analysis shows this principle to be sound, there are a lot of similar cases where "Run past all builders" (as we might name this rule) is the perfect play by a considerable margin. This applies not only to the bold D and \$ openings, but also to S, Z, and P openings, ²⁰ as well as to the reply rolls of 62 and 63:

```
41$-63 [R S11]

41$-63 [R S2]

43Z-64 [R P15]

51$-62 [R S14]

51$-63 [R S33]

51$-62 [R S37]

51$-63 [R S48]

52$-63 [R 19]

54D-63 [R 13]

54D-64 [R S20 P42]

61P-64 [R P29]
```

It seems to be a great idea across the board and, even if not right in all cases, ²¹ loses a lot less equity. However, it should neither be used with 54 nor, obviously, with 53. Hence our addition to the rule set will be:

• 62R, 63R, 64R past all builders

By now, all semi-whoppers on our not-to-do list have been covered (except for one, that will be dealt with, fittingly, in Exception). I did not list them all in detail, because this information can be found in Mistakes by "Ugly Cygnet". We have rules in place that help us avoid them by pointing to alternative moves instead. Let us gather together our findings now in a small library.

4.3.8 Library

Our library should of course contain the correct opening moves for money games and match play (we are using the same moves for both). For reply moves, it should include the defaults along with the alternatives required to avoid semi-whoppers. Finally, it should provide moves for cases where a default move is blocked by an opponent's point-making opening move:

Roll	Default	Alternative
21, 41, 51	\$	S
31, 42, 53, 61	P	P
32	D	S
43	D	Z
52, 54	S	D
62, 63	S	R
64	S (opening), P (reply)	R
65	R	D

Some random observations:

- The "Beautiful Cygnet" played rather boring moves ("Splits, anyone?"), which proved to be a robust recommendation for many reply situations. Our "Ugly Cygnet" tried far more glamorous moves than its stepsibling, but literally got badly hit: as we have seen, it innocently ran into way too many double shots. Slots against an opening split are thrilling, but terribly dangerous.²² So the reckless replies needed refinement and the sexy stuff was rightly reserved for adults. Our cygnet needed more care to grow up successfully, a clear case of parental neglect.
- The library and our rule set have been designed to cover the following table of 30 opening moves without causing any semi-whoppers:

Roll	Moves
21, 41, 51	\$, S
31, 42, 53, 61	P
32	D, S, Z
43	D, Z, S, U
52, 54	S, D
62	S, R, \$
63	S, R
64	S, P, R
65	R

This is a somewhat larger set of opening moves than our library uses for the reply moves, but the additions are definitely moves that we will encounter in real life: 32Z, 43S, 43U, or 62\$.

• The potential of our library is huge. If we had perfect rules to decide between the default and alternative move in the 289 non-hitting cases

(the library should be used only for these), the total equity loss would be a mere 0.2753, caused by 24 mistakes, most of them below 0.01 equity loss and only 5 of them errors. Compare this with the performance of our "Ugly Cygnet", who, granted, also had to deal with hitting situations and had to survive without alternative moves! But why are there still mistakes at all, even though we assumed perfect decisions between default and alternative moves? Because the library in these 24 cases simply does not contain the right move. In a sense, it is like a beautiful walled garden, perhaps even with a pond for ducks or swans. However, it should also have designated exits for escaping in case of trouble. This will be covered in the next section.

• From eXtreme Gammon's opening book, our library contains only *strate-gically diverse* moves that are no errors. This means that 32D and 32S are part of it, but 32Z is not, because it is too similar to 32S:

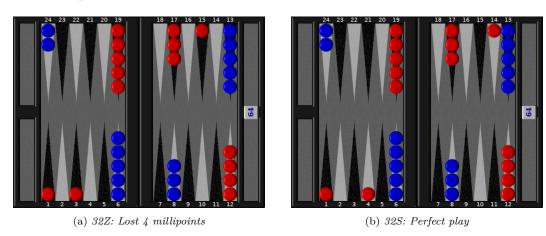


Figure 22: Library: No strategic difference

Likewise, it includes 43D and 43Z, but not 43S, which is too similar to 43Z. However, all of 64S, 64P, and 64R *are* in our library, because splitting, pointing, and running are very different plays. In this way our small library can be expected to cover a lot of strategically different situations and hopefully gives a lot of bang for our ducks.

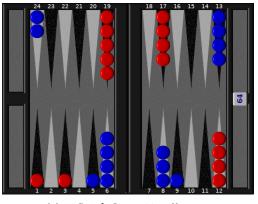
4.3.9 Exception

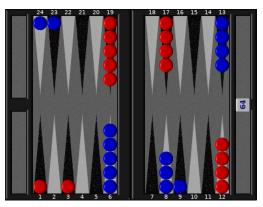
After analyzing the semi-whoppers and devising rules to avoid them by picking the alternative move from our library, there is one case left that needs to be treated as an exception:

52S-41 [D U6 H43 S56 ... \$103]

This Nacbrac shows that the default move (\$) results in a whopper and the alternative move (S) still in a semi-whopper. It is the *only* non-hitting situation where we cannot play from our library and have to leave our walled garden.

What is causing the trouble? Our opponent has split her back checkers, so our default 41\$ will give her a double shot on our slot. Sounds familiar? Well, yes, but this time it is different: our rule mandating safe play gives us no way out, because the alternative 41S is safer, but still a semi-whopper. Our outfield checker is still exposed to a single direct shot. This is not always a problem, e.g., we often tolerate these shots against splits to the bar-point, but this position is unique in that there is a direct shot from our home board to our outer board (more succinct: "direct shot across the bar") that cannot be avoided by either play from our library. This is causing the exception.





(a) 52S-41: Lost 103 millipoints

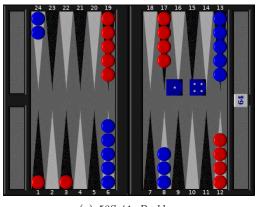
(b) 52S-41S: Lost 56 millipoints

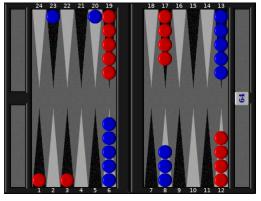
Figure 23: Exception: Default or alternative?

Let us shut down this freak show by giving a proper answer to this exciting reply problem. In fact, most beginners would find the perfect solution and calmly play 13/8 ("D" in Nactation). However, since this exception has a close, but less bad sibling,

32Z-41 [U H5 S11 D14 ... \$96]

my recommendation is to play two checkers up from the rear, 24/23 24/20 ("U" in Nactation), which summed over both cases loses a little bit less equity.





(a) 52S-41: Problem (b) 52S-41U: Solution

Figure 24: Exception: Think from scratch!

Our last addition to our "Ugly Cygnet" will thus be:

• Leave no direct shot across the bar: play 24/23 24/20

4.3.10 Candidate rule set

The following list gathers the usable genes of our "Ugly Cygnet" and refines or augments them according to the rules we found by analyzing the "not-to-do list" of semi-whoppers. You will see that compared to the rule set for "Ugly Cygnet" the sequence of items has changed slightly: I did not want to hop back and forth between hitting and pointing moves, but rather preferred a clean partitioning between hitting, after hitting, and no hitting (default moves), see the outline of a rule set. What needed to be added, though, are the rules that decide which move from the library should be played: the default or the alternative. The rest may look like fine tuning or cosmetic change, but be warned: Fiddling with the sequence of prioritized rules takes you into dangerous waters. All kinds of weird and unforeseen interaction effects may emerge from the depths and initially promising changes may produce zero or even negative results. I can tell you from my experience (lots of it . . .) that checking all 450 non-doublet reply moves, after a slight change of structure, is no fun.

(Default moves: 65R, P, 43D, 32D, 51\$, 41\$, 21\$, S)

- 1. Hitting deep
 - With 65: 13/7* 6/1*
 - With 54 or 52: After 62S or 64S, play $24/x 6/1^*$

- With 43 or 32: 6/3*
- With 41: 6/2*/1*
- 2. Hitting high
 - With non-pointing roll: Hit above 3
 - With pointing roll: Hit above "12 roll", then Point On Head
- 3. After hitting
 - 20/14
 - 5 checkers on your mid-point? Play 13/x to unstack, else 24/x
- 4. Check default move (not needed for pointing rolls or 65)
 - Leaves direct shot across the bar? Play 24/23 24/20
 - Has opponent split or made a point? Play 43Z or S
- 5. Running
 - With 62R, 63R, 64R: Run past all builders
- 6. Play default move

(If blocked: R, D, \$)

4.3.11 Results

Criterion	Result
Correct moves	381 (85 %)
$Mistakes \leq 0.05$	69 (15%)
Mistakes > 0.05	0 (0%)
Total equity loss	1.007
Maximum equity loss per mistake	0.046
Average equity loss per mistake	0.015
Average equity loss per move	0.002

Our "Pretty Duck" performs much better than both young birds on all accounts: it makes fewer mistakes, avoids all semi-whoppers and suffers much less equity loss (total and maximum). As before, the Nacbracs for *all* mistakes can be found in Mistakes by "Pretty Duck".

5 Reply (doublets)

This section on doublets is much shorter than the one for non-doublets. Not many design iterations were necessary for the rule set, and therefore I will present only our "Pretty Duck". The reason for this is that the rules given in Bagai's article cannot be shortened much without giving rise to massive mistakes. With my goal to avoid all semi-whoppers, I could only afford some minor omissions from his (again!) outstanding material.

However, after staring long and hard at the doublet replies (to the point of getting "square eyes" from "Snake Eyes"), I was able to completely re-organize them in the same way as the non-doublet replies, i. e., within my framework of "Hitting", "After hitting", and "No hitting" with default and alternative moves taken from a library. To my surprise, all doublets fit into this scheme, in theory enabling an extremely compact and consistent rule set covering 180 replies. In practice, though, perfect play against our set of 30 openings can be achieved with only two different moves each for replies with 66 or 55. Hence, I have omitted these trivial cases from the general framework, which would have meant unnecessary (mental) overhead over the board for these two rolls.

With a doublet as reply roll, only in some cases a checker is moved alone, typically to hit. Most plays just shift or make points, ²³ hence the main question is: "Which points?" This is sometimes influenced by strategic considerations, but more often dictated by blots lying around after our opponent's opening move. Similar to non-doublets, when we hit twice ("kill"), we can afford to hit deeper in our home board than with a single hit. In contrast to non-doublets, it is much easier with a doublet to do good things on both sides of the board. Our range for action is still limited by other factors, though: we should rarely waste the full roll for shifting a point or use more than two numbers for hitting a "fly shot". 24 After hitting a blot, we should often keep two of our four numbers in reserve to make an additional inner point. However, this should not be done automatically, because the default moves are very strong, even when they do not fortify our home board. They are default for a reason, and, with a blot on the bar as collateral damage, they become only stronger. You will see that most of these ideas can be found in our rule set below. More detailed instructions will be postponed to the summary, to avoid duplication.

5.1 Candidate rule set

- 1. With 66: Play 24/18(2) 13/7(2). Play 13/7(2) 8/2(2) if blocked.
- 2. With 55: Play 13/3(2). Play 8/3(2) 6/1*(2) if it hits.
- 3. Look up library move:

Roll	Default	Alternative	If blocked
11	8/7(2) 6/5(2)	After non-6 split: 24/22 6/5(2)	
22	13/11(2) 6/4(2)		
33	24/21(2) 13/10(2)	After \$ or D: 24/21(2) 8/5(2) After 63R: 13/10*(2) 6/3(2)	8/5(2) 6/3(2)
44	24/20(2) 13/9(2)		13/5(2)

4. Hitting

- Hit highest blot above point 2 (use at most 2 numbers)
- If successful: Hit another blot above point 1 (use 1 further number)

5. After hitting

- Cover blot below point 8
- Complete/Play library move (if still possible)
- With 44: Point on 20 (or 9, worse)
- With other rolls: Point in home board

6. No hitting

• Play library move

5.2 Results

Criterion	Result
Correct moves	162 (90%)
$Mistakes \leq 0.05$	18 (10%)
Mistakes > 0.05	0 (0%)
Total equity loss	0.284
Maximum equity loss per mistake	0.045
Average equity loss per mistake	0.016
Average equity loss per move	0.002

The performance of our "Pretty Duck" for doublets is comparable to the non-doublets regarding the percentage of mistakes and the maximum or average equity loss. As before, the information on *all* mistakes can be found in the appendix. Like for non-doublets, all semi-whoppers are successfully avoided. This delivers on my third, and final promise. Mission accomplished.

6 Summary

Some prefer to memorize tables like Backgammon Opening Replies, others dig long and hard for simple heuristics that make them less smart²⁵. Our "Ugly Cygnet", smuggled as a duck's egg (with a small "genetic mutation") into a swan's nest, was rightly vilified by its stepsibling, the "Beautiful Cygnet". Now mature, and having absorbed some lessons on how to survive on a dangerous backgammon board, it has become a fine specimen of a different kind of waterfowl: with a more compact constitution, our "Pretty Duck" is not as beautiful as the majestic swan, but very tasty if prepared properly.

We are now ready for the following tasks:

- 1. Play all opening moves in money games with an equity loss of at most 0.02
- 2. Play all opening moves for all possible scores of a 5-point match with an equity loss of at most 0.05
- 3. Play all reply moves in money games with an equity loss of at most 0.05

The basis for this is our library of default and alternative reply moves (which uses a little bit of Nactation). For your convenience, it is repeated here from the main part of the article:

Roll	Default	Alternative
21, 41, 51	\$	S
31, 42, 53, 61	P	P
32	D	S
43	D	\mathbf{Z}
52, 54	S	D
62, 63	S	R
64	S (opening), P (reply)	R
65	R	D

In case you have any doubts whether your interpretation of the following rules for the opening and reply moves is correct, you can always consult the XG database: If the move you came up with after following the rules loses more equity than promised above, you have obviously done something wrong. Each recommended reply move that loses equity at all is listed in the appendix. So if the listed move and your move differ, you have also done something wrong. And if your move loses zero equity, you have obviously done everything right.

6.1 Opening

For money games and match play, use the default opening moves from the library:

6.2 Reply (non-doublets)

The complete rule set for non-doublet reply moves is as follows:

(Default moves: 65R, P, 43D, 32D, 51\$, 41\$, 21\$, S)

- 1. Hitting deep
 - With 65: 13/7* 6/1*
 - With 54 or 52: After 62S or 64S, play $24/x 6/1^*$
 - With 43 or 32: 6/3*
 - With 41: 6/2*/1*

- 2. Hitting high
 - With non-pointing roll: Hit above 3
 - With pointing roll: Hit above "12 roll", then Point On Head
- 3. After hitting
 - 20/14
 - 5 checkers on your mid-point? Play 13/x to unstack, else 24/x
- 4. Check default move (not needed for pointing rolls or 65)
 - Leaves direct shot across the bar? Play 24/23 24/20
 - Has opponent split or made a point? Play 43Z or S
- 5. Running
 - With 62R, 63R, 64R: Run past all builders
- 6. Play default move

(If blocked: R, D, \$)

We start with the rules for the **hitting** cases. The first item deals with some special cases in which hitting deeper than usual is required. The second item covers the standard cases where we hit higher than a particular point. For a non-pointing roll this limit is 3. For a pointing roll, we calculate it by subtracting our roll from 12. So with 61 rolled, we hit above 5 (= 12 - 6 - 1), for 53 rolled, we hit above 4 (= 12 - 5 - 3). Always hit as often and as high as possible. The third item deals with the remainder of our roll **after hitting** and requires continuing to run into the outfield, or aiming for 4 checkers on the mid-point otherwise.

In the subsequent rules we turn to our library for the non-hitting cases. Item 4 (not needed for pointing rolls or 65) will **check** a **default move** to ensure it is not too bold. It will direct us to safer alternative moves in these cases. Item 5 helps us to decide when (even with a pointing roll) it is safer to "get the hell out of there" and **run** into the outfield to escape priming or attacking threats. Note that running is advised only for the "R" moves from our library, i. e., 62R, 63R, and 64R. In contrast, 53R and 54R are not part of the library for a good reason: they are never the perfect play and often result in semi-whoppers. Finally, if there are no "contraindications", we **play** the **default move**. If it is blocked, we switch to another move from the library.

For players who dislike long lists, there is a colorful chart for the above rules.

6.3 Reply (doublets)

The complete rule set for doublet reply moves is as follows:

- 1. With 66: Play 24/18(2) 13/7(2). Play 13/7(2) 8/2(2) if blocked.
- 2. With 55: Play 13/3(2). Play 8/3(2) 6/1*(2) if it hits.
- 3. Look up library move:

Roll	Default	Alternative	If blocked
11	8/7(2) 6/5(2)	After non-6 split: 24/22 6/5(2)	
22	13/11(2) 6/4(2)		
33	24/21(2) 13/10(2)	After \$ or D: 24/21(2) 8/5(2) After 63R: 13/10*(2) 6/3(2)	8/5(2) 6/3(2)
44	24/20(2) 13/9(2)		13/5(2)

4. Hitting

- Hit highest blot above point 2 (use at most 2 numbers)
- If successful: Hit another blot above point 1 (use 1 further number)

5. After hitting

- Cover blot below point 8
- Complete/Play library move (if still possible)
- With 44: Point on 20 (or 9, worse)
- With other rolls: Point in home board

6. No hitting

Play library move

66 or 55 are easy cases and treated first, since they will be played in an either/or way and do not need a more sophisticated step-by-step approach.

Look up the correct **library move** for the other rolls. However, it will not be played immediately, but rather should be treated as a candidate move and kept in mind for the subsequent list items.

Hitting is to be carried out under certain restrictions to ensure that we will not waste valuable numbers by hitting fly shots or too deep in our home board. In

particular, it *excludes* pointing on head, so we must not hastily move additional numbers on autopilot. The clean-up of this potentially blotted board will be done next.

After hitting our opponent will be on the bar, thus it is crucial to cover the blots at least in our home board. It cannot be bad either to cover a blot on our bar-point, because this checker is exposed to a direct shot from our point 1. Also, having the bar-point makes for a nice blocking point with two checkers behind it. If, at this step in our recipe, we notice that we are still on our way to the result we would have gotten anyway by playing the library move (and are just walking a ceremonial detour), by all means we should play it, whether it is the default one or the alternative. Library moves are good, even more so with the opponent on the bar. If, on the other hand, we notice that we have strayed too far from the well-trodden library path, we should put our remaining numbers to other good use, e.g., by making a point. Here, 44 is the odd (or rather even?) man out: we are better off making the higher points (preferably 20, otherwise 9), not points in our home board as with the other replies.²⁶

No hitting, no fun. Play a boring default or alternative move from the library (or the one from the last column of the table in case the others are blocked). Whatever you looked up in step 3.

There is also a flowchart for the doublets.

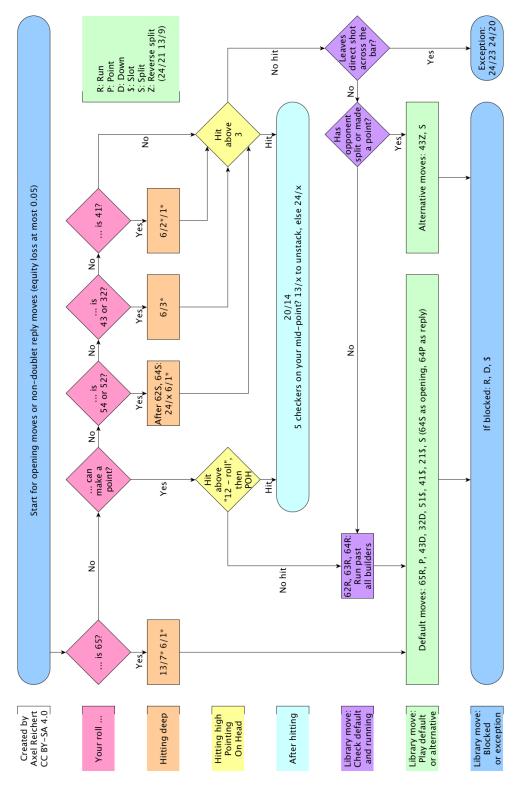
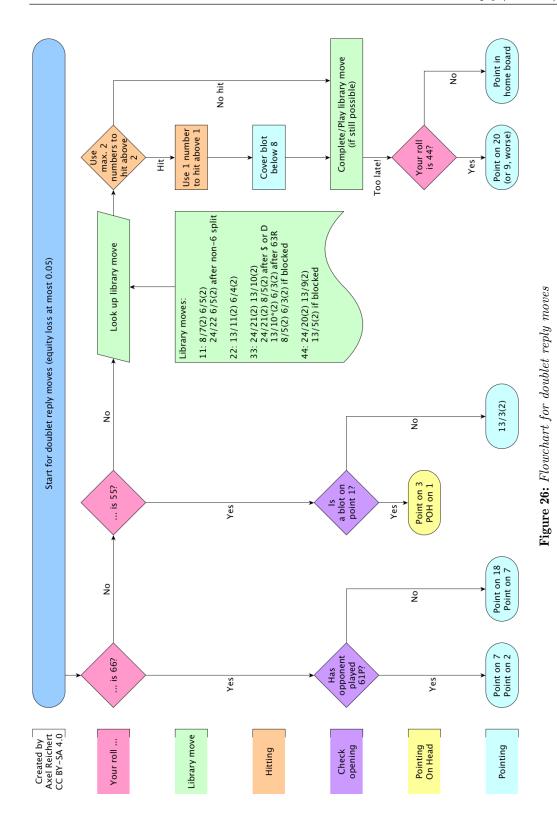


Figure 25: Flowchart for non-doublet reply moves



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7 Acknowledgements

Many thanks go to Jeremy Bagai, whose article got me started on this topic, and to Nack Ballard, who helped in a brainstorming session on bgonline.org with ideas to make my former rule set both simpler and more accurate. Peter Durrans, fortunately for you and me a native speaker, became my proofreader by serendipity. We had a tough nit-picking contest, and I am not certain who won it between the two of us. His hawk-eyed attention to detail was highly unusual and very welcome in this aquatic environment. The clear profiteer has been this article: some of his wordings were so well-rounded that the laying of the resulting smooth egg (to keep the waterfowl theme swimming one last time) must have been as much a pleasure for mother duck as our collaboration was for me.

8 Appendix

The appendix lists the Nacbracs of *all* mistakes made by our three waterfowl in non-doublet situations. Especially for the younger birds the mistakes are sometimes so large that no rollouts are available from the XG database. In these cases I have augmented the Nacbracs with millipoints provided by GNU Backgammon 3-ply analysis. An ellipsis ("...") denotes that some information is missing or not needed. This is not really relevant, because these moves usually are so bad (more than 0.07 equity loss) that additional data would not help in the search for a better rule set, but I wanted to mention this technicality for your reference.

There are also some Nacbracs with more exotic Nactation, such as the numbers 1, 2, and 4 or the letters "O" and "V". Hopefully I got these right, but I am definitely no expert in Nactation (the XG database uses the standard notation). These exotic symbols are not explained in the main text to avoid confusion for players who are new to this shorthand notation. The full Nactation documentation by its inventor provides plenty of details.

The mistakes for doublet rolls are only given for our "Pretty Duck" as explained in Reply (doublets). I have not used Nactation for the doublets, because the standard notation is easier to follow.

8.1 Mistakes by "Beautiful Cygnet"

Situation	Nacbrac
(Non-doublet)	
21\$-43	[U S3]
21\$-63	[R Z9 U17 S29]
21S-41	[K S35]
21S-43	[X Z1]
31P-32	[Z S9]
31P-64	[R S16 P29]
32D-21	[\$ S23]
32D-32	[D Z2 S11]
32D-41	[\$ S18]
32D-43	[D Z19]
32D-51	[\$ S3]
32S-62	[X H8]
32Z-32	[K X4 H11 S62]
	[U H5 S11]
32Z-43	[X H11 S93]
32Z-64	[H R26 S40 P55]
41\$-21	[\$ S23]
41\$-43	[U S18]
41\$-51	[\$ S11]
41\$-63	[R S11]
41\$-64	[R Z57]
41S-32	[Z S2]
41S-41	[K S47]
41S-43	[X Z33]
41S-54	[X S1]
41S-63	[R S2]
42P-32	[Z D27]
42P-43	[S D3]
43D-21	[\$ S65]
43D-32	[Z D7 S37]
43D-41	[\$ S41]
43D-43	[D Z41]
43D-51	[\$ S31]
43D-64	[R S1 P71]
43S-41	[X H8]

Situation	Nacbrac
(Non-doublet)	
43S-43	[X H15]
43S-53	[H P75]
43S-61	[X P4]
43U-43	[X K24]
43U-53	[H P9]
43U-54	[S H0]
43U-63	[4 H44]
43U-65	[S R20]
43Z-32	[X H2]
43Z-64	[R P15]
51\$-21	[\$ S21]
51\$-51	[\$ S7]
51\$-52	[\$ S0]
51\$-62	[R S14]
51\$-63	[R 425 S33]
51\$-64	[R Z53]
51S-21	[U \$9 D21 S22]
51S-41	[K D15 U31 S51]
51S-43	[U D4 S7 Z10]
51S-62	[R S37]
51S-63	[R S48]
52D-21	[\$ S66]
52D-32	[D S23]
52D-41	[\$ S46]
52D-43	[D Z30]
52D-51	[\$ S33]
52D-52	[D \$25 S29]
52D-54	[D R13 S21]
52D-63	[R S3]
52S-32	[H S8]
52S-41	[D U6 H43 S56]
52S-43	[H X3 U46 S63 Z100]
52S-52	[D S6]
52S-63	[R S19]
52S-64	[H R13 P15]
53P-43	[D Z2]
54D-21	[\$ S79]

Situation	Nacbrac
(Non-doublet)	
54D-32	[D S7]
54D-41	[\$ W57 S81]
54D-43	[D Z26]
54D-51	[\$ S74]
54D-52	[D \$16 S31]
54D-54	[D R1 S15]
54D-63	[R S13]
54D-64	[R S20 P42]
54S-53	[H P42]
61P-21	[\$ S24]
61P-32	[Z S1]
61P-43	[U Z3]
61P-64	[R P29]
62\$-21	[\$ S33]
62\$-32	[\$ D11 S13]
62\$- 51	[\$ S17]
62\$- 52	[\$ D8 S10]
62\$-63	[\$ S3]
62\$-64	[R Z50]
62R-21	[U V24 W27 \$31 S36]
	[U S7]
62R-62	[\$ Z2 U12 R17 S26]
62R-63	[Z S22]
62S-31	[P X40 H53]
62S-32	[U W1 S1]
62S-52	[X S8]
62S-54	[X S32]
62S-65	[K D50]
63R-62	[\$ S10]
63R-64	[S P21]
63S-31	[P X46 H60]
63S-52	[X S1]
63S-65	[K D60]
64P-32	[D Z15 S21]
64P-43	[D Z32]
64P-54	[D S29]
64P-62	[\$ S1]

Situation	Nacbrac
(Non-doublet)	
64R-41	[\$ W18 S18]
64R-43	[S Z14]
64R-51	[\$ S6]
64S-31	[X P2 H32]
64S-41	[X H6]
64S-52	[X S56]
64S-54	[X S56]
64S-65	[K D75]
65R-21	[\$ S25]
65R-52	[D S15]

8.2 Mistakes by "Ugly Cygnet"

Situation	Nacbrac
(Non-doublet)	
21\$-21	[S \$0]
21\$-32	[S Z23 D23]
21\$-43	[U S3]
21\$-51	[S \$18]
21\$-63	[R Z9 U17 S29]
21S-21	[S U3 V37 \$46]
21S-32	[S U23 Z31 D38]
21S-41	[K S35 U47 X48 D52 \$95]
21S-43	[X Z1 D17]
21S-51	[S U48 X62 \$76]
31P-21	[S V38 U53 \$65]
31P-32	[Z S9 U22 D37]
31P-41	[S D57 \$83]
31P-43	[Z D28]
31P-51	[S U43 \$88]
31P-64	[R S16 P29]
32S-51	[S U43 \$55]
32S-62	[X H8]
32Z-21	[S H4 V45 U54 D71 \$75]
32Z-32	[K X4 H11 S62 Z74 D90]
32Z-41	[U H5 S11 D14 154 286 \$96]

Situation	Nacbrac
(Non-doublet)	
32Z-43	[X H11 Z93 S97 D124]
32Z-51	[S U1 X21 140 \$78]
32Z-64	[H R26 S40 P55]
41\$-32	[S Z31 D40]
41\$-43	[U S18]
41\$-63	[R S11]
41\$-64	[R Z57]
41S-21	[S U15 \$24]
41S-32	[Z S2 D27]
41S-41	[K S47 X52 U57 D59 \$82]
41S-43	[X Z33 D43]
41S-51	[S U43 X50 \$67]
41S-54	[X S1]
41S-63	[R S2]
42P-21	[S \$30]
42P-32	[Z D27]
42P-41	[S \$38]
42P-43	[S D3]
42P-51	[S \$57]
43D-32	[Z D7]
43S-41	[X H8]
43S-43	[X H15]
43S-53	[H P75]
43S-61	[X P4]
43U-43	[X K24]
43U-53	[H P9]
43U-63	[4 H44]
43U-65	[S R20]
43Z-32	[X H2]
43Z-51	[S X24 U41 \$54]
43Z-64	[R P15]
43Z-64	[R P15]
51\$-32	[S Z2 42 D16]
51\$-52	[4 S0]
51\$-62	[R S14]
51\$-63	[R 425 S33]
51\$-64	[R Z53]

Situation	Nacbrac
(Non-doublet)	
51S-21	[U \$9]
51S-32	[S D4]
51S-41	[K D15 U31 S51 X52 W77 \$78]
51S-43	[U D4]
51S-51	[S U32 \$39]
51S-62	[R S37]
51S-63	[R S48]
52D-63	[R S3]
52S-21	[S H18 U28 \$33]
52S-32	[H S8 K19 X23 D43]
52S-41	[D U6 H43 S56 \$103]
52S-43	[H X3 U46 S63 D108]
52S-51	[S U27 X58 \$59]
52S-52	[D S6]
52S-63	[R S19]
52S-64	[H R13 P15]
53P-21	[S \$3]
53P-32	[S D12]
53P-41	[S \$22]
53P-51	[S \$27]
54D-52	[D \$16 S31]
54D-63	[R S13]
54D-64	[R S20 P42]
54S-53	[H P42]
61P-32	[Z S1 U23 D41]
61P-41	[S \$19]
61P-43	[U Z3 S26 D46]
61P-51	[S \$37]
61P-64	[R P29]
62\$-32	[4 D11]
62\$-52	[\$ D8 S10]
62\$-63	[4 S3]
62\$-64	[R Z50]
62R-21	[U V24 W27 \$31]
62R-32	[U S7 D98]
62R-51	[S \$22]
62R-62	[\$ Z2 U12 R17 S26]

Situation	Nacbrac
(Non-doublet)	
62R-63	[Z S22]
62S-31	[P X40 H53]
62S-32	[U W1 S1 D86]
62S-43	[Z U10 S41 D87]
62S-52	[X S8]
62S-54	[X S32]
62S-65	[K D50]
63R-41	[S \$2]
63R-51	[S \$15]
63R-62	[\$ S10]
63R-64	[S P21]
63S-31	[P X46 H60]
63S-32	[S W2 Z3 U3 D71]
63S-43	[Z S6 U35 H55 D85]
63S-52	[X S1]
63S-65	[K D60]
64P-54	[D S29]
64P-62	[\$ S1]
64R-43	[S Z14 D30]
64S-31	[X P2 H32]
64S-32	[S H18 Z30 D84]
64S-43	[Z S19 W30 U35 H58 D82]
64S-52	[X S56]
64S-54	[X S56]
64S-65	[K D75]
65R-32	[S D20]
65R-41	[S \$4]
65R-43	[Z S8 D15]
65R-51	[S \$3]
65R-52	[D S15]

8.3 Mistakes by "Pretty Duck"

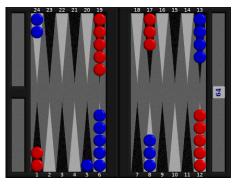
Situation	Nacbrac
(Non-doublet)	
21\$-21	[S \$0]
21\$-32	[S U23 D23]
21\$-51	[S \$18]
21\$-63	[R Z9 U17 S29]
21S-43	[X Z1]
31P-32	[Z S9]
31P-64	[R P29]
32S-62	[X H8]
32Z-32	[K X4 H11]
32Z-43	[X H11]
41\$-32	[S Z31 D40]
41S-32	[Z S2]
41S-43	[X Z33]
41S-54	[X S1]
42P-32	[Z D27]
42P-43	[S D3]
43D-32	[Z D7]
43S-41	[X H8]
43S-43	[X H15]
43S-61	[X P4]
43S-64	[P R12]
43U-43	[X K24]
43U-54	[S H0]
43U-63	[4 H44]
43U-64	[P O19 H24]
43U-65	[D R20]
43Z-32	[X H2]
51\$-32	[S Z2 \$2 D16]
51\$-52	[\$ S0]
51S-21	[U \$9 D21 S22]
51S-43	[U D4 S7 Z10]
52D-52	[D \$25 S29]
52D-54	[D R13 S21]
52S-41	[D U6]
52S-52	[D S6]

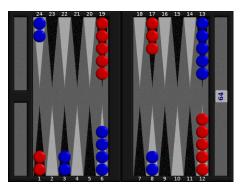
Situation	Nacbrac
$({\it Non-doublet})$	
52S-62	[S R5]
53P-43	[D Z2]
54D-52	[D \$16 S31]
54D-54	[D R1 S15]
54S-64	[P R46]
61P-21	[\$ S24]
61P-32	[Z S1]
61P-43	[U Z3]
62\$-32	[\$ D11]
62\$-52	[\$ D8 S10]
62\$-63	[\$ S3]
62R-21	[U V24 W27 \$31 S36]
62R-32	[U S7]
62R-51	[S \$22]
62R-62	[\$ Z2 U12 R17 S26]
62R-63	[Z S22]
62S-32	[U W1 S1]
63R-41	[S \$2]
63R-51	[S \$15]
63R-62	[\$ S10]
63R-64	[S P21]
63S-52	[X S1]
64P-32	[D Z15 S21]
64P-43	[D Z32]
64P-54	[D S29]
64P-62	[\$ S1]
64R-43	[S Z14 D30]
64S-31	[X P2]
64S-41	[X H6]
65R-32	[S D20]
65R-41	[S \$4]
65R-43	[Z S8 D15]
65R-51	[S \$3]
65R-52	[D S15]

Situation (Doublet)	Perfect move	Played move	Equity loss (millipoints)
(Doublet)			(mmpomts)
31P-11	$24/22 \ 6/5(2)$	8/7(2) 6/5(2)	3
31P-22	24/22(2) 6/4(2)	13/11(2) 6/4(2)	14
41\$-11	24/20*	8/7(2) 6/5(2)	8
41S-22	24/22(2) 6/4(2)	13/11(2) 6/4(2)	11
42P-22	24/22(2) 6/4(2)	13/11(2) 6/4(2)	18
43D-22	24/22(2) 6/4(2)	13/11(2) 6/4(2)	29
43D-44	24/20(2) $13/9(2)$	24/16*13/9(2)	9
52D-33	8/5(2) 6/3(2)	24/21(2) 8/5(2)	31
54D-22	24/22(2) 6/4(2)	13/11(2) 6/4(2)	34
54D-33	24/21(2) 6/3(2)	24/21(2) 8/5(2)	17
62\$-33	8/5(2) 6/3(2)	24/21(2) 8/5(2)	3
62R-11	24/226/5(2)	8/7(2) 6/5(2)	9
62R-33	8/5(2) 6/3(2)	24/21(2) $13/10(2)$	20
63R-11	24/226/5(2)	8/7(2) 6/5(2)	3
64P-22	24/22(2) 6/4(2)	13/11(2) 6/4(2)	6
64R-33	8/5(2) 6/3(2)	24/21(2) $13/10(2)$	13
64S-22	24/16*	13/11(2) 6/4(2)	45
64S-44	24/16*(2)	24/16* 13/9(2)	11

9 Notes

- 1. This includes sessions played with or without JACOBY rule.
- 2. "Error" means an equity loss of more than 0.02. This follows the definition/default setting in eXtreme Gammon. Equity loss in general (more than 0.00) is termed "Mistake". If "equity" is a foreign concept to you, see the next footnote.
- 3. "Whopper" means an equity loss of more than 0.10 (or 100 millipoints), see the glossary at Backgammon Galore!. This is already quite a serious mistake. The following example shows an opening roll of 53 played so badly that it is almost exactly a whopper:





(a) 53\$: Lost 105 millipoints

(b) 53P: Perfect play

Figure 27: No beginner would slot, but correctly make point 3!

Put differently (in money terms): if you are playing for 1 euro/dollar per point, every whopper costs you more than 10 cents. A "semi-whopper" at these stakes thus corresponds to (all synonymously) 5 cents, an equity loss of 0.05, or 50 millipoints.

4. If the perfect play for replying with a particular doublet *does not* require hitting, the following table should steer you clear of any whoppers:

Roll	Default	If blocked
11	8/7(2) 6/5(2)	
22	13/11(2) 6/4(2)	
33	$24/21(2) \ 13/10(2)$	8/5(2) 6/3(2)
44	24/20(2) $13/9(2)$	13/5(2)
55	13/3(2)	
66	$24/18(2) \ 13/7(2)$	13/7(2) $8/2(2)$

However, if perfect play *requires* you to hit, you are on your own. You must pay a price for your laziness.

- 5. This is neither the only one avoiding semi-whoppers nor the "best" one (in the sense of smallest total equity loss over all moves and scores). The best one would use 41S and 51S. But as we will see later in this article, it will simplify things massively to treat all slots with a 1 alike.
- 6. The worst opening moves resulting from this list are:

Move	Match score	Equity loss
64S	4-away/2-away	0.0441
41\$	4-away/1-away Post-Crawford	0.0428
51\$	4-away/1-away Post-Crawford	0.0421

Now if you know your opening moves and match scores already in some detail, at 4-away/2-away you will play the move with the highest gammon chances, which is, of course, 64P. Likewise, you will not slot in even-away Post-Crawford games: you will never make that point, because your opponent will use her free drop.

- 7. Which is why I wrote this article in the first place. However, my recommended ploy will backfire once your opponents have read this article.
- 8. According to current "hard facts" for money games with JACOBY rule, these are the equity losses caused by intentional non-optimal openings for these rolls:

Best move	Second-best move	Equity loss
32S	32D	0.0132
41S	41\$	0.0153
51S	51\$	0.0157

We will see the potential equity gains caused by unintentional non-optimal replies for these rolls later in the article.

- 9. ... or were not played, because we had hit erroneously.
- 10. Besides hitting, which will be covered by other rules anyway, 64S is perfect play only for 63R-64, where making point 2 loses 21 millipoints over the split. 64R is best in 5 non-hitting cases, 64P in 8.
- 11. Neither 64P nor 64R will be free of semi-whoppers in a 5-point match: 64P fails for 2-away/4-away and 2-away/2-away, while 64R fails for 4-away/2-away. So 64S is the only possible solution for matches, if we want a *single* list of openings.
- 12. "Nactation" is a portmanteau word combining "NACK" (BALLARD) and "action notation".
- 13. Why not always hit on point 3, no matter with which number? We then could merge "Hit on 3" (generalized from "6/3*") and "Hit above 3" to "Hit above 2". Beware, there is only one further case where we gain a couple of millipoints, but plenty of others where we lose much more. This is not a good idea. And why not hit on point 3 at least with 63, which also contains a 3? Because it loses 29 millipoints after 52S and is a triple whopper after 32Z (there was a blot on point 15, dummy).
- 14. I lied to you, at least kind of. There *is* one further case, and it was already shown at the start of this section:

32Z-32 [K X4 H11 S62 Z74 D90]

You can see that the hitting play as advocated by our first new rule for hitting deep and rule 4 (the part after hitting) loses 11 millipoints to the kill. In my opinion this is neither something you should worry about nor is it worth an additional rule or exception.

- 15. This looks like 4 cases, not 5. However, the default 63S-52S loses less than 1 millipoint against "hit and split", so I omitted the 63S opening for an easier rule, especially since 63S-54 of course requires hitting with 24/15*.
- 16. Trying to get rid of the POH clause by changing the rule to "Hit above '11 roll'" or "Hit down to (including) '12 roll'" will not work: 41S-64 will then happily hit on point 2, but our rule for the remainder of the roll will then dictate playing the other number down from the mid-point. Both 13/9 8/2* and 13/7 6/2* are almost triple whoppers.

17. While the "Rule of 12" kills 10 birds with one stone and prevents an equity loss of 0.765, it is not perfect:

```
43S-61 [X P4]
43U-64 [P 019 H24]
43Z-64 [R P15 X32 H40]
64S-31 [X P2]
```

18. Besides the Nacbracs in the main text, the largest mistake I found was a mere 15 millipoints:

```
43S-43 [X H15]
```

This is not even an error.

- 19. Why is the fourth slot play (21\$-64) missing in the Nacbracs? Because then our rule 1 mandates to scoop up the second blot in the outfield anyway, and 21\$-64K is of course correct.
- 20. It definitely does *not* apply to R openings: there is no builder to "Run past", since no checker was played down from the mid-point.
- 21. The largest mistake is:

```
54S-64 [P R46]
```

This is still below semi-whopper size.

- 22. By the way, always slotting with a 1 as default play might have seemed a little bold, but it resulted in a much less diverse range of mistakes that could be covered completely with surprisingly few additional rules. So our consistency had little, if any, equity cost, but big conceptual benefits.
- 23. To be precise, with perfect play according to the XG database, 175 out of 180 doublet replies make an additional point or at least increase the number of made *home board* points (if a point is shifted or another point is given up).
- 24. This is confirmed by the XG database: The are only two correct fly shots (after 41\$-11 we should hit with $24/20^*$ and after 64S-22 with $24/16^*$) and eight "non-trivial" shifts (13/5(2) after 31P as well as 13/3(2) is "semi-forced") using up all four numbers. These shifts, however, all point on head to make the move worthwhile.
- 25. At least "less accurate". Far less accurate than BAGAI's rule set. But in the end, it might be smart for non-world-class players to sacrifice some accuracy for a smaller set of rules and exceptions that we can actually memorize and hence will use.
- 26. A possible explanation for this curious fact is that 44 with its 16 pips is quite a good racing roll, so we should get going.